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Scattering of Ultrashort Electromagnetic Pulses by a Free Electron in the Nonrelativistic Limit

ASTAPENKO V.A. AND SAKHNO S.V.

Moscow Institute of Physics and Technology (State University)

ABSTRACT: The paper is devoted to theoretical study of ultrashort electromagnetic pulses scattering on free electron in nonrelativistic limit. Two types of pulses are considered: (1) with carrier frequency and (2) without it. Analytical expressions for total scattering probability are derived. It is shown that in the first case the probability dependence upon pulse duration has inflecting point while in the second case this dependence is quadratic in all range of pulse duration.

INTRODUCTION

The paper considers the process of scattering of ultrashort electromagnetic pulses (USP) of different shapes by a free electron: analytical expressions for the scattering probability during the action of a pulse were obtained, corresponding dependences were plotted. Calculations were carried out for a corrected Gaussian pulse as well as for sine and cosine wavelet pulses without carrier frequency [1].

METHOD OF CALCULATION

Let us obtain a general expression for the total probability of scattering of an ultrashort electromagnetic pulse during the USP action [2].

At first, we will consider scattering of an electromagnetic field pulse by a spherical target as a consequence of induction of an alternative dipole moment in the target by the electric field of a scattered wave.

In the wave zone, for radiation of an alternative dipole moment (in view of a delay) the following equality is true [3]:

$$\mathbf{E}(t,\mathbf{r}) = \frac{1}{c^2\, r}\left[\left[\ddot{\mathbf{d}}(t - r/c)\mathbf{n}\right]\mathbf{n}\right] \quad \mathbf{n} = \frac{\mathbf{r}}{r} \tag{1}$$

For the second derivative of the dipole moment of the target that scatters a pulse, we have the following expansion into a Fourier integral:

$$\ddot{\mathbf{d}}(t-r/c) = -\int_{-\infty}^{\infty} \omega^2 \, \mathbf{d}(\omega) \exp\left(-i\,\omega\,(t-r/c)\right)\frac{d\omega}{2\pi} =$$
$$= -\int_{-\infty}^{\infty} \exp\left(-i\,\omega(t-r/c)\right)\omega^2\,\alpha(\omega)\,\mathbf{E}_i(\omega)\,\frac{d\omega}{2\pi} \tag{2}$$

Here in the second equality it is taken into account that for a spherically symmetric target

$$\mathbf{d}(\omega) = \alpha(\omega)\,\mathbf{E}_i(\omega), \tag{3}$$

where $\alpha(\omega)$ is the dynamic polarizability of the spherically symmetric target and

$$\mathbf{E}_i(\omega) = \int_{-\infty}^{\infty} \exp(i\,\omega t)\,\mathbf{E}_i(t)\,dt \tag{4}$$

is the Fourier transform of an electric field strength pulse scattered by the target.

From (1) - (2), the equality follows for an electric field strength pulse scattered in the direction $\mathbf{n}$:

$$\mathbf{E}_{sc}(t,\mathbf{r}) = \frac{1}{c^2\,r}\left[\left[-\int_{-\infty}^{\infty} \exp\left(-i\,\omega(t-r/c)\right)\omega^2\,\alpha(\omega)\,\mathbf{E}_i(\omega)\,\frac{d\omega}{2\pi}\;\mathbf{n}\right]\mathbf{n}\right]. \tag{5}$$

For simplicity, we assume a scattered pulse to be linearly polarized:

$$\mathbf{E}_i(\omega) = \mathbf{e}_i\,E_i(\omega), \tag{6}$$

where $\mathbf{e}_i$ is the real unit polarization vector in a scattered pulse. Then from (5) we obtain:

$$\mathbf{E}_{sc}(t,\mathbf{r}) = -\frac{\left[\mathbf{n}\left[\mathbf{e}_i\,\mathbf{n}\right]\right]}{c^2\,r}\int_{-\infty}^{\infty} \exp\left(-i\,\omega(t-r/c)\right)\omega^2\,\alpha(\omega)\,E_i(\omega)\frac{d\omega}{2\pi} \tag{7}$$

It is easy to show that the magnitude of the triple vector product in the formula (7) is

$$\left|\left[\mathbf{n}\left[\mathbf{e}_i\,\mathbf{n}\right]\right]\right| = \left|\mathbf{e}_i - \mathbf{n}\left(\mathbf{e}_i\,\mathbf{n}\right)\right| = \sin\theta \tag{8}$$

where θ is the angle between the vectors $\mathbf{n}$ and $\mathbf{e}_i$. Then from the formula (6) we have the expression in scalar writing for the strength of the electric field in a scattered pulse:

$$E_{sc}(t,\mathbf{r}) = \frac{\sin\theta}{c^2\,r}\int_{-\infty}^{\infty} \exp\left(-i\,\omega(t-r/c)\right)\omega^2\,\alpha(\omega)\,E_i(\omega)\frac{d\omega}{2\pi} \tag{9}$$

The obtained expression makes it possible to calculate the shape of a scattered pulse for various types of targets [4].

We proceed from the formula (9) for the strength of the electric field in a scattered wave in the wave zone ($r >> \lambda$) obtained in the preceding paragraph within the framework of classical electrodynamics. Using this formula, we will calculate the energy ΔE_{sc} of radiation scattered into a complete solid angle during the action of an electromagnetic pulse. For this purpose, we will use the expression for the radiation intensity in a transverse electromagnetic wave propagating in vacuum:

$$I_{sc}(t,\mathbf{r}) = \frac{c}{4\pi}\left|E_{sc}(t,\mathbf{r})\right|^2 \tag{10}$$

Thus for the energy of a scattered electromagnetic pulse we have:

$$\Delta \mathrm{E}_{sc} = \int d\Omega \int_{-\infty}^{+\infty} dt\, r^2 I_{sc}(t,\mathbf{r}) = \frac{c}{4\pi}\int d\Omega \int_{-\infty}^{+\infty} dt\, r^2 \left|E_{sc}(t,\mathbf{r})\right|^2 \tag{11}$$

Substituting here the explicit expression for the electric field strength E_{SC} (t, $\mathbf{r}$) from the formula (9), we find:

$$\Delta \mathrm{E}_{sc} = \frac{1}{2(2\pi)^3 c^3}\int \sin^2\theta\, d\Omega \int_{-\infty}^{+\infty} dt \int_{-\infty}^{\infty} \exp\left(-i\omega(t-r/c)\right)\omega^2 \alpha(\omega) E_i(\omega)\, d\omega \times$$
$$\times \int_{-\infty}^{\infty} \exp\left(i\omega'(t-r/c)\right)\omega'^2 \alpha^*(\omega') E_i^*(\omega')\, d\omega'. \tag{12}$$

Integrating in the right-hand side of this equation with respect to time, we obtain $2\pi\delta(\omega - \omega')$ under the sign of integration with respect to the frequencies ω and ω'. The delta-function "removes" integration with respect to one of frequencies, say, ω', as a result of which we obtain:

$$\Delta \mathrm{E}_{sc} = \frac{1}{3\pi c^3}\int_{-\infty}^{\infty} \omega^4 \left|\alpha(\omega)\right|^2 \left|E_i(\omega)\right|^2 d\omega \tag{13}$$

Let us use the expression for the cross-section of radiation scattering by a spherically symmetric target that is integrated with respect to the angle:

$$\sigma_{sc}(\omega) = \frac{8\pi}{3}\left|\frac{\omega^2}{c^2}\alpha(\omega)\right|^2 \tag{14}$$

Then the expression for the scattered energy can be rewritten as

$$\Delta \mathrm{E}_{sc} = \frac{c}{4\pi^2}\int_{0^-}^{\infty} \sigma_{sc}(\omega)\left|E_i(\omega)\right|^2 d\omega \tag{15}$$

Hence for the scattered energy spectrum we have:

$$\frac{d\Delta \mathrm{E}_{sc}}{d\omega} = \frac{c}{4\pi^2}\sigma_{sc}(\omega)\left|E_i(\omega)\right|^2 \tag{16}$$

and for the spectral probability we find:

$$\frac{dW_{sc}}{d\omega} = \frac{1}{\hbar\omega}\frac{d\Delta \mathrm{E}_{sc}}{d\omega} = \frac{c}{4\pi^2 \hbar\omega}\sigma_{sc}(\omega)\left|E_i(\omega)\right|^2 \tag{17}$$

Thus for the total probability of scattering of an ultrashort electromagnetic pulse we finally obtain the expression:

$$W_{sc} = \frac{c}{4\pi^2}\int_0^\infty \sigma_{sc}(\omega)\frac{|E_i(\omega)|^2}{\hbar\omega}d\omega \tag{18}$$

This formula coincides (accurate to replacement of the radiation scattering cross-section by the photoabsorption cross-section) with the expression for the *photoabsorption* probability during the action of an ultrashort pulse that was obtained earlier within the framework of the quantum-mechanical approach in the work [2]. It should be noted that (18) is applicable if $W_{SC} \leq 1$, by implication of the process probability.

Let us consider the process of radiation scattering by a free electron in the nonrelativistic limit, that is, when the radiation energy is much less than the rest energy of an electron:

$$\hbar\omega << m_e c^2 . \tag{19}$$

In this case it is possible to use the following expression for the Thomson cross-section of photon scattering by an electron that is integrated with respect to the angle [3]:

$$\sigma^{(Th)} = \frac{8\pi\ r_e^2}{3}, \tag{20}$$

where $r_e = \dfrac{e^2}{m_e c^2}$ is the electron classical radius.

Thus within the framework of the approximation (19) the scattering cross-section is independent on frequency and in the atomic units we will use in further calculations looks as follows:

$$\sigma_{sc} = \frac{8\pi}{3\cdot c^4}, \tag{21}$$

where c = 137 is the velocity of light in the atomic system of units.

As scattered electromagnetic pulses, we will consider the following pulses:

$$E_{CGP}(t) = \mathrm{Re}\left[-iE_0 \frac{\left(1+it/\omega\tau^2\right)^2 + 1/(\omega\tau)^2}{1+1/(\omega\tau)^2}\exp\left(\frac{-t^2}{2\tau^2}\right)\exp(i\omega\tau + i\varphi)\right] \tag{22}$$

$$E_{\sin}(t,\tau) = \frac{\sqrt{2}E_0}{\sqrt[4]{\pi}}\frac{t}{\tau}\exp\left(\frac{-t^2}{2\tau^2}\right), \tag{23}$$

$$E_{\cos}(t,\tau) = \frac{2E_0}{\sqrt{3}\sqrt[4]{\pi}}\left(1-\left[\frac{t}{\tau}\right]^2\right)\exp\left(\frac{-t^2}{2\tau^2}\right), \tag{24}$$

where E_0 is the peak value of the electric field strength, ω is the radiation carrier frequency, τ is the USP duration, φ is the initial phase.

These are a so-called corrected Gaussian pulse (CGP) (22), sine (23) and cosine (24) wavelet pulses respectively. Note that two last pulses have no carrier frequency.

A feature of a CGP is the absence of a constant component in its spectrum, which distinguishes it from a traditional Gaussian pulse. It should be noted that the presence of such a constant component in the spectrum in the limit of ultrashort pulse durations results in the nonphysical nature of obtained results.

The Fourier transform for a CGP looks as follows:

$$E_{CGP}(\omega',\omega,\tau,\varphi) = E_0\tau\sqrt{\frac{\pi}{2}}\frac{\omega'^2\tau^2}{1+\omega^2\tau^2}\left[\exp(-i\varphi-(\omega-\omega')^2\frac{\tau^2}{2}) - \exp(i\varphi-(\omega+\omega')^2\frac{\tau^2}{2})\right] \quad 25$$

where ω' is the current frequency and ω is the carrier frequency of radiation. It should be noted that for this expression the condition $E_{CGP}(\omega' = 0) = 0$ is true, which confirms the absence of a constant component in the CGP spectrum.

For sine and cosine wavelet pulses without carrier frequency, the Fourier transforms have the following expressions [1]:

$$E_s(\omega) = 2i\sqrt[4]{\pi}\,\omega\tau^2 E_0 \exp\left(\frac{-\omega^2\tau^2}{2}\right), \quad (26)$$

$$E_c(\omega) = 2\sqrt{\frac{2}{3}}\sqrt[4]{\pi}\,\omega^2\tau^3 E_0 \exp\left(\frac{-\omega^2\tau^2}{2}\right) \quad (27)$$

Substituting (21) and (26) in the formula for the total probability of the scattering process during the action of a pulse (18), we obtain:

$$W_s(\tau) = \frac{c\sigma_e}{4\pi^2}\int_0^\infty \omega\tau^4 E_0^2 \exp\left(\frac{-\omega^2\tau^2}{2}\right)d\omega$$

Replacing $x = \omega\tau$ and taking into account the fact that the $\int_0^\infty x\exp\left(-x^2\right)dx = \frac{1}{2}$, we obtain the final result:

$$W_s(\tau) = \frac{c\sigma_e\tau^2 E_0^2}{4\pi}\int_0^\infty x\exp\left(-x^2\right)dx = \frac{4}{3\sqrt{\pi}}\frac{E_0^2}{c^3}\tau^2 \quad (28)$$

Making similar rearrangements and using the expression (23), we obtain the dependence for the total probability of the scattering process during the action of a cosine wavelet pulse:

$$W_c(\tau) = \frac{8\sqrt{\pi}}{9}\frac{E_0^2}{c^3}\tau^2 \quad (29)$$

In case of a CGP, for times of interest to us, the approximate equality for the squared absolute value of the Fourier transform of the strength of the electric field in a pulse is true:

$$\left|E_{CGP}(\omega',\omega,\tau)\right|^2 = E_0^2\tau^2\frac{\pi}{2}\left(\frac{\tau^2\omega'^2}{1+\tau^2\omega^2}\right)^2 \exp\left(-(\omega-\omega')^2\tau^2\right) \quad (30)$$

In this case the expression for the total probability of the scattering process during the action of a pulse (18) will take the following form:

$$W_{CGP}(\omega,\tau) = \frac{E_0^2}{3c^3\omega^2} F_c(\omega\tau), \tag{31}$$

where $F_c(x) = \frac{x^2}{2(1+x^2)^2}\left\{(1+x^2)\exp(-x^2) + \sqrt{\pi}x(1+erf(x))\left(x^2+\frac{3}{2}\right)\right\}$.

The inflection point of this function is equal to $x_i \cong 1.3$.

Let us consider the limit of ultrashort pulses, when $\omega\tau \ll x_i$. In this case the expression (31) can be expanded into a Taylor series with respect to $\omega\tau$, which gives

$$W_{CGPS}(\omega,\tau) \approx \frac{E_0^2}{3c^3}\tau^2\left(1+\frac{3\sqrt{\pi}}{2}\omega\tau\right). \tag{32}$$

On the contrary, in case of long multicycle pulses ($\omega\tau \gg x_i$) the expression (31) can be transformed to the following form:

$$W_{CGPL}(\omega,\tau) \approx \sqrt{\pi}\frac{E_0^2}{3c^3}\frac{\tau}{\omega^3}. \tag{33}$$

From the expressions (32), (33) it is seen that in the ultrashort limit the scattering process probability depends on the pulse duration as τ^2, whereas in case of long multicycle pulses this dependence is proportional to τ (is linear).

DISCUSSION OF RESULTS

Presented in Fig. 1 are the dependences of the total probability of scattering of sine and cosine wavelet pulses as functions of the pulse duration, $E_0 = 1$ at. u.

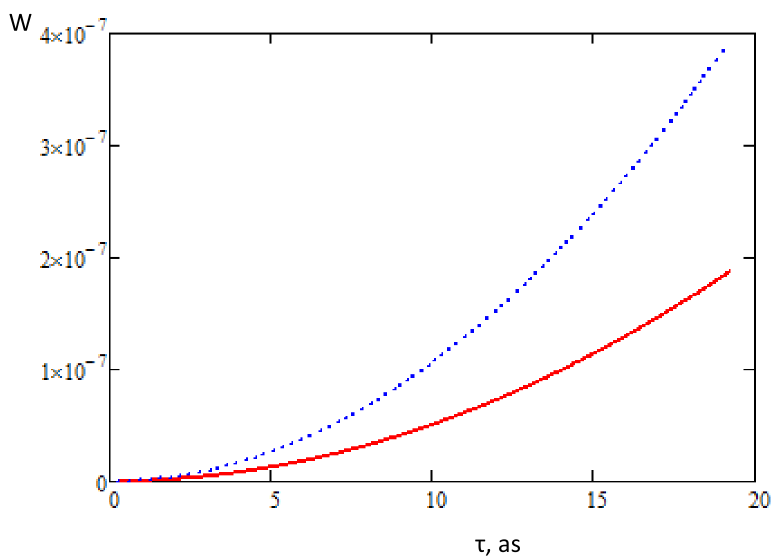

Fig. 1. The dependence of the total probability of USP scattering by a free electron on the pulse duration: solid curve - sine wavelet, dotted curve – cosine wavelet

As follows from the expressions (28) and (29), quadratic dependences are seen on the plot, and in case of a cosine wavelet pulse, due to the higher coefficient at τ^2 in the expression (29), the plot of this dependence (the dotted curve) is above the sine wavelet plot (the solid curve). It should also be noted that in case of wavelet pulses the total probability of scattering depends only on the duration of these pulses τ.

Presented in Fig. 2 is the dependence of the total probability of scattering of an ultrashort pulse (CGP) by a free electron on the pulse duration at different carrier frequencies of radiation ω.

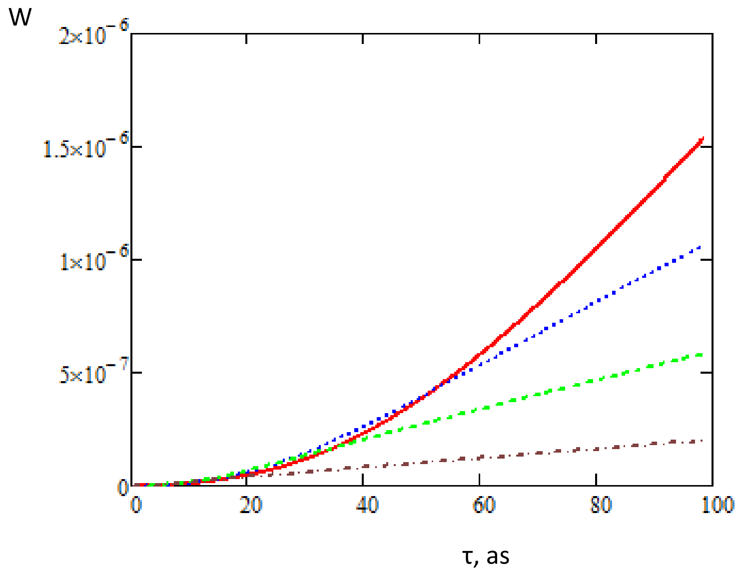

Fig. 2. The dependence of the total probability of USP scattering by a free electron on the pulse duration for a CGP at different carrier frequencies: dash-and-dot curve – 0.11 eV, short dashes - 0.037 eV, dotted curve – 0.018 eV, solid curve – 0.009 eV

From Fig. 2 it follows that at long enough pulse durations τ the plots go to the linear dependence, and with decreasing carrier frequency of a pulse the slope of the straight line increases.

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