I R A M P
6(2), Dec. 2015, pp. 67-71

ISSN: 2229-3159

Scattering of Ultrashort Electromagnetic Pulses by a two-level System

V. A. ASTAPENKO AND S.V. SAKHNO

Moscow Institute of Physics and Technology

ABSTRACT: The paper is dedicated to the study of scattering of ultrashort pulses (USP) of different shapes by a two-level system (TLS). In case of a high quality factor of a TLS and for sufficiently short pulses, an analytical expression for the scattering probability during the action of a pulse was obtained. Scattering of pulses with a carrier frequency (Gaussian and superfluorescence pulses) and without a carrier frequency (sine and cosine wavelet pulses) was considered. The dependences of the scattering probability on the pulse duration for different USP types were calculated and analyzed.

The rapid development of the technology for generation of ultrashort electromagnetic pulses (USP) in a wide spectral range and with specified parameters necessitates the development of the theory of their interaction with a substance [1].

A two-level system (TLS) with a dipole-allowed transition is the simplest and at the same time the most important quantum object for construction of the theory of electromagnetic radiation interaction with a matter.

The specificity of TLS excitation by short laser pulses was considered in the works [2-4]. The present paper is devoted to consideration of scattering of USP of different types by a TLS with a high quality factor (Q factor).

Let us consider Rayleigh scattering of an ultrashort electromagnetic pulse by a TLS. The TLS parameters: ω_0 is the eigenfrequency, f_0 is the oscillator strength, γ is the damping constant. Q factor is equal to the ratio $Q = \omega_0 / \gamma$.

The probability of scattering of an USP during its action is given by the expression [5]:

$$W = \frac{c}{(2\pi)^2} \int_0^\infty \sigma(\omega') \frac{|E(\omega', \tau)|^2}{\hbar \omega'} d\omega', \tag{1}$$

where $\sigma(\omega')$ is the scattering cross-section, $E(\omega', \tau)$ is the Fourier transform of the strength of the electric field in an USP, c is the velocity of light, τ is the USP duration.

The spectral cross-section of Rayleigh scattering of radiation is

$$\sigma(\omega) = \frac{8\pi}{3} \left(\frac{\omega}{c}\right)^4 |\alpha(\omega)|^2, \tag{2}$$

where

$$\alpha(\omega) = \frac{e^2}{m} \frac{f_0}{\omega_0^2 - \omega^2 - i\omega\gamma} \tag{3}$$

is the dipole polarizability of the TLS in case of homogeneous broadening.

Substituting the formula (3) in (2), we find the following expression for the cross-section of radiation scattering by the TLS:

$$\sigma(\omega)=\frac{8\pi}{3}\left(\frac{\omega}{c}\right)^4\left(\frac{e^2}{m}\right)^2\frac{f_0^2}{\left(\omega_0^2-\omega^2\right)^2+\omega^2\gamma^2}. \tag{4}$$

Then we will consider a case of a high Q factor of a TLS, when

$$\omega_0 >> \gamma. \tag{5}$$

besides, we assume that the spectral width of an USP $\Delta\omega$ is much more than the width of a transition line in the TLS:

$$\Delta\omega >> \gamma. \tag{6}$$

The inequality (6) is characteristic of USP, the duration of which is $\tau << 1/\gamma$. Really, since $\Delta\ \omega \geq 1/\ \tau$, the inequality (6) is fulfilled.

In view of (5) - (6), the expression for the scattering cross-section can be reduced to the form

$$\sigma(\omega)\cong\frac{4\ \pi^2}{3}\left(\frac{e^2}{m\ \ c^2}\right)^2\frac{f_0^2\omega_0^2}{\gamma}\delta(\omega-\omega_0), \tag{7}$$

where $\delta(w)$ is the Dirac delta function. In derivation of the approximate equation (7), representation of the delta function in terms of the Lorentz function in the limit of a small damping constant was used: $\gamma \to 0$.

Substituting the scattering cross-section (7) in the formula (1) and carrying out integration with the use of the delta function, we obtain

$$W(\tau)\cong\frac{1}{3}\left(\frac{e^2}{m\ \ c^2}\right)^2\frac{f_0^2\omega_0\ c}{\hbar\gamma}\left|E(\omega_0,\tau)\right|^2. \tag{8}$$

Thus in the case under consideration the dependence of the USP scattering probability on the pulse duration is reduced to the τ-dependence of the squared absolute value of the Fourier transform of the electric field in an USP.

It should be noted that a similar result is obtained for the probability of excitation of a radiative transition under the USP action in case of a narrow spectral line [6].

At first, let us consider a case of an USP with a Gaussian envelope, for which we have approximately

$$\left|E_G(\omega',\tau)\right|^2\approx\frac{\pi}{2}\tau^2E_0^2\exp\left[-(\omega-\omega')^2\tau^2\right], \tag{9}$$

where ω is the carrier frequency of an USP, E_0 is its amplitude. From (8) - (9) it is easy to obtain the expression for the pulse duration, at which the scattering probability reaches the maximum value:

$$\tau_{\max}^{(G)}=\frac{1}{\left|\omega-\omega_0\right|}. \tag{10}$$

It is obvious that (10) is meaningful for $\omega \neq \omega_0$. In the opposite case $\omega = \omega_0$ the maximum of the function $W(\tau)$ is absent: the scattering probability increases quadratically with lengthening of a pulse $W(\tau)\propto\tau^2$.

For cosine and sine wavelet pulses without a carrier frequency, we have

$$\left|E_c\left(\omega',\tau\right)\right|^2 \propto \omega'^4 \tau^6 E_0^2 \exp\left[-\omega'^2\tau^2\right], \tag{11}$$

$$\left|E_s\left(\omega',\tau\right)\right|^2 \propto \omega'^2 \tau^4 E_0^2 \exp\left[-\omega'^2\tau^2\right]. \tag{12}$$

Then from (8) we can obtain:

$$\tau_{\max}^{(c)} = \frac{\sqrt{3}}{\omega_0} \tag{13}$$

and

$$\tau_{\max}^{(s)} = \frac{\sqrt{2}}{\omega_0}. \tag{14}$$

It should be noted that for wavelet pulses without a carrier frequency there is no resonance case ($\omega = \omega_0$), therefore the scattering probability always decreases with increasing t. Really, the maximum of the spectrum (11) - (12) corresponds to the frequency $\omega_{\max} \propto 1/\tau$, so at $\tau \to \infty$ the scattering probability tends to zero because then $\omega_{\max} \neq \omega_0$ and $\Delta\omega \to 0$.

Given below are the results of calculations by the formula (8) for the normalized probability of scattering

$$\tilde{W} = \frac{W}{E_0^2} \tag{15}$$

as a function of the pulse duration.

As a TLS, we will consider the transition in a magnesium atom ${}^1S_0 \to {}^1P_1^{(o)}$ with the oscillator strength f_0 = 1.9 and the eigenfrequency $\omega_0 \cong 0.16$ a.u. ($6.6\cdot10^{15}$ s^{-1}). If it is assumed that damping of the oscillator connected with the TLS is caused by spontaneous decay of the upper level, the ratio is $\omega_0/\gamma \approx 10^8$, that is, the assumption of a high Q factor is fulfilled to a high accuracy.

Presented in Fig. 1 is the probability of scattering of a Gaussian pulse for different detunings of the carrier frequency of a pulse from the eigenfrequency (TLS). It is seen that with decreasing detuning the maximum of the probability is shifted to the region of longer pulses according to the formula (10) and increases in amplitude.

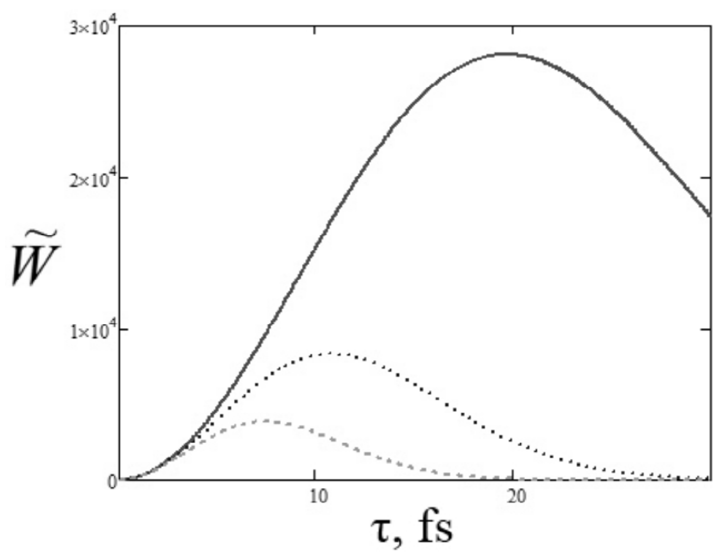

Figure 1: The probability of scattering of a Gaussian USP by a TLS for different carrier frequencies: solid curve - ω = 0.161 a.u., dotted curve - ω = 0.162 a.u., dashed curve - ω = 0.163 a.u.; abscissa in fs

Fig. 2 demonstrates a similar dependence for scattering of sine and cosine wavelet pulses. In this case, since the carrier frequency is absent, the dependences of the scattering probability on the pulse duration for specified TLS parameters are of universal character.

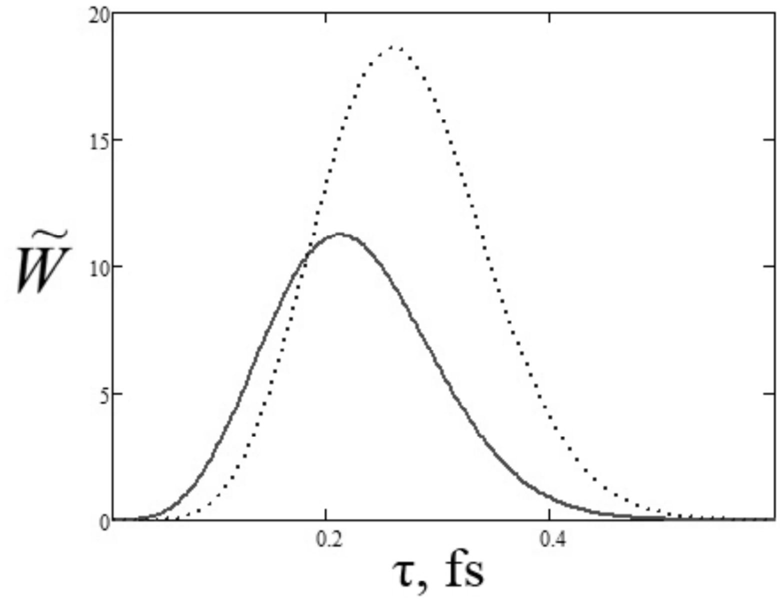

Figure 2: The probability of scattering of wavelet pulses by a TLS: solid curve - sine wavelet, dotted curve - cosine wavelet; abscissa in fs

According to the formulas (13), (14), the maximum of the probability for a sine pulse is somewhat shifted to the region of shorter durations in comparison with the case of a cosine pulse, and the amplitude at the maximum is lesser.

In case of a superfluorescence pulse we have

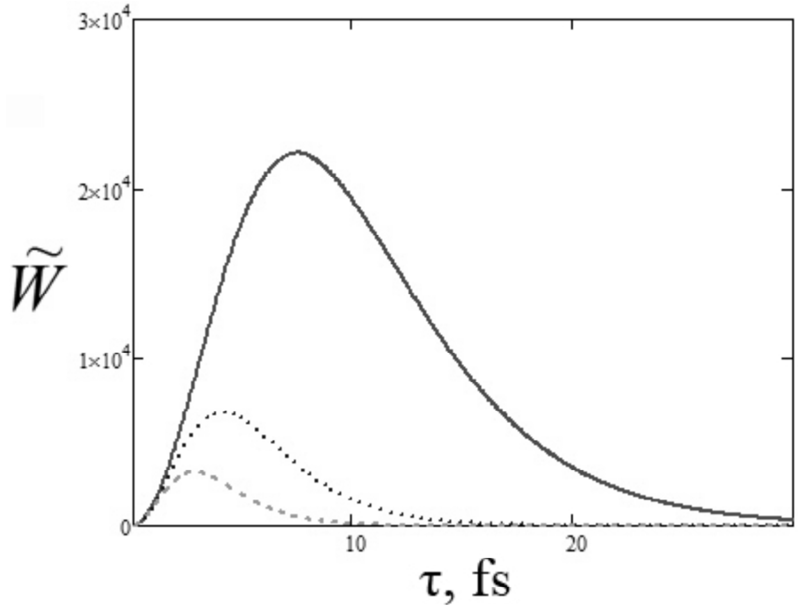

Figure 3: The probability of scattering of a superfluorescence pulse by a TLS for different carrier frequencies: solid curve - w = 0.161 a.u., dotted line - w = 0.162 a.u., dashed curve - w = 0.163 a.u.; abscissa in fs

$$\left|E_{SF}\left(\omega',\tau\right)\right|^2 \approx \pi^2\tau^2 E_0^2 \sec h^2\left[\pi\left(\omega-\omega'\right)\tau\right] \quad (16)$$

and

$$\tau_{\max}^{(SF)} \cong \frac{1.2}{\pi\ \left|\omega-\omega_0\right|}, \quad (17)$$

that is, as in the case of a Gaussian pulse, the duration at the maximum of the scattering probability is inversely proportional to the detuning of the USP carrier frequency from the TLS eigenfrequency.

From Figs. 1 and 3 it follows that for USP with a carrier frequency the dependences of the scattering probability on the pulse duration are close to each other.

As seen from comparison of Figs. 1-3, in case of USP with a carrier frequency the maximum of the scattering probability lies in the femtosecond range of USP durations (for a specified TLS eigenfrequency), while for wavelet pulses without a carrier frequency this maximum falls in the subfemtosecond range. The said fact corresponds to the formulas (10), (13), (14), and (17) for the pulse duration at the maximum of the scattering probability.

References

[1] V. Astapenko *Interaction of ultrafast electromagnetic pulses with matter* Springer Briefs in Physics. Heidelberg, New York, Dordrecht, London. 2013. – 94 p.

[2] N. V. Bordyug, V. P. Krainov, Dynamic resonances in ultra-short laser pulses. // Laser Physics Letters 4 418–420 (2007).

[3] N. V. Bordyug, V. P. Krainov, Dynamics of two-level systems irradiated by sub-one-cycle laser pulses. // Laser Physics Letters 4 674–677 (2007).

[4] M. G. Arustamyan, V. A. Astapenko, Phase control of two-level system excitation by short laser pulses.// Laser Physics, 18, 10131-1036 (2008).

[5] V. A. Astapenko, Simple formula for photoprocesses in ultrashort electromagnetic field. // Physics Letters A 374 1585–1590 (2010).

[6] V. A. Astapenko, V. A. Bagan, Features of excitation of two-level system by short nonresonance laser pulses.// Journal of Physical Science and Application 3 269–277 (2013).