Manifestations of Langmuir Solitons in Satellites of Dipole-forbidden Spectral Lines of Helium, Lithium, and of the Corresponding Ions

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ABSTRACT: We show that in the case of Langmuir solitons in plasmas, the peak intensity of the satellites of dipole-forbidden spectral lines of helium, lithium, and of the corresponding ions can be significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves. This distinctive feature of satellites under Langmuir solitons allows distinguishing them from non-solitonic Langmuir waves and seems to be the best way to diagnose Langmuir solitons in plasmas.

In paper [1] there was calculated analytically the shape of satellites of dipole-forbidden lines in a spectrum *spatially-integrated* through a Langmuir soliton (or through a sequence of Langmuir solitons separated by a distance L). The Langmuir solitons have the following form in space [2]:

$$F(x, t) = E(x) \cos \omega_p t, \qquad E(x) = E_0/\mathrm{ch}(x/\lambda), \qquad \lambda << L. \tag{1}$$

Here

$$\omega_p = \omega_{pe} - 3T_e/(2m_e \omega_{pe} \lambda^2), \tag{2}$$

where ω_{pe} is the plasma electron frequency. For diagnosing solitons it is necessary not only to find experimentally an electric field oscillating at the frequency $\sim \omega_{pe}$, but also to make sure that the spatial distribution of the amplitude corresponds to the formfactor $E(x)$ from (1).

Under any quasimonochromatic electric field, dipole-forbidden spectral lines of helium, lithium, and of the corresponding ions can exhibit satellites. In cases of a relatively large separation between the forbidden and allowed lines, or a relatively weak amplitudes, the intensities of the far (+) and near (–) satellites are $S_\pm = a_\pm E^2(x)$, where $a_\pm$ does not depend on x.

The spatially-integrated profile of the satellites, calculated with the allowance for the quadratic shift of their frequencies $bE^2(x)$, where b does not depend on x, has the form:

$$S_\pm(\Delta\omega) = (1/L) \int_{L/2}^{L/2} dx \, a_\pm \, E^2(x) \, \delta[f(x)], \quad f(x) = \Delta\omega \pm \omega_p - bE^2(x). \tag{3}$$

After calculating the integral in Eq. (7.3), in paper [1] it was obtained:

$$S \pm (\Delta\omega) = (\lambda/L)[a_\pm/(2|b|)]/[1 - (\Delta\omega \pm \omega_p)/(bE_0^2)]^{1/2}. \tag{4}$$

The profiles $S_\pm(\Delta\omega)$, formally calculated by Eq. (4), have singularities at $\Delta\omega \pm \omega_p = bE_0^2$. From the physical point of view, for obtaining a finite result at $\Delta\omega \pm \omega_p = bE_0^2$, it is necessary to replace the δ-function in Eq. (3) by a real profile, e.g., by the Lorentz profile representing the dynamical Stark broadening by electrons and by some ions:

$$\delta[f(x_0)] \rightarrow (1/\pi)\gamma/\{\gamma^2 + [f(x_0)]^2\}, \tag{5}$$

where x_0 is the root of the equation $f(x) = 0$. Using the Taylor expansion of $f(x)$ at $x = x_0$ and taking into account that the first derivative df/dx vanishes at $x = x_0$, the right side of Eq. (5) can be approximated as follows:

$$(1/\pi)\gamma/\{\gamma^2 + [d^2f(x_0)/dx^2]^2(x - x_0)^4\}. \tag{6}$$

Then using the integral

$$(1/\pi)\int_{-\infty}^{\infty} dz\,\gamma/(\gamma^2 + z^4) = 1/(2\gamma)^{1/2}, \tag{7}$$

we find

$$S_{\text{soliton}}^{\text{max}} = S_{\pm}(\Delta\omega = \pm\,\omega_p - bE_0^2) = [\lambda a_{\pm}/(2^{1/2}\pi Lb^{1/2})]E_0^2/\gamma^{1/2}, \tag{8}$$

where $S_{\text{soliton}}^{\text{max}}$ is the peak intensity of the satellites in the case of solitons.

In the case of non-solitonic Langmuir waves, the peak intensity of the satellites is (see, e.g., book [3]):

$$S_{\text{nonsoliton}}^{\text{max}} = a_{\pm}E_0^2/(\pi\gamma). \tag{9}$$

So, for the ratio of the peak intensity of the satellite under solitons to the peak intensity of the same satellite under nonsolitons we get

$$S_{\text{soliton}}^{\text{max}}/S_{\text{nonsoliton}}^{\text{max}} = [\lambda/(2^{1/2}L)][\gamma/(bE_0^2)]^{1/2} \gg 1 \tag{10}$$

for the typical situation where $\gamma \gg bE_0^2$, i.e., where the dynamical Stark width γ is much greater than the quadratic shift bE_0^2 of the satellites.

Thus, in the case of Langmuir solitons, the peak intensity of the satellites can be significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves. This distinctive feature of satellites under Langmuir solitons allows distinguishing them from non-solitonic Langmuir waves and seems to be the best way to diagnose Langmuir solitons in plasmas.

Finally we note that for a more general case, where both the dynamical Stark broadening and the Doppler broadening are taken into account, the δ-function in Eq. (3) should be substituted by the Voigt profile resulting in

$$S_{\text{soliton}}^{\text{max}}/S_{\text{nonsoliton}}^{\text{max}} \sim [\Delta\omega_{\text{Voigt}}/(bE_0^2)]^{1/2} \gg 1, \tag{11}$$

where $\Delta\omega_{\text{Voigt}}$ is the halfwidth of the Voight profile. Again, in the typical situation where

$\Delta\omega_{\text{Voigt}} \gg bE_0^2$, the peak intensity of the satellites is significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves.

Finally we mention that Hannachi *et al.* [4] performed simulations for finding the effect of Langmuir solitons on the hydrogen Ly_α line. The effect was an additional broadening. However, even at the low electron density $N_e = 10^{14}$ cm^{-3}, the effect was very small compared to the Stark broadening by plasma microfields. Moreover, the additional broadening rapidly diminished with the increase of N_e, so that there would be practically no additional broadening at $N_e > 10^{15}$ cm^{-3}. Therefore, it seems that the results by Hannachi *et al.* [4] could not be useful for the experimental diagnostics of Langmuir solitons.

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