TOPOLOGY PRELIMINARY EXAMINATION

Thursday, June 3, 1999

Solve eight out the following problems. All problems will be weighted equally. Use a separate sheet for each problem you solve.

Circle the numbers of the eight problems you chose:

Problem 1. Suppose that X is a metric space. Prove that the collection of all open balls in X is a basis.

Problem 2. Give an example of a Hausdorff space which is not regular.

Problem 3. Prove that every compact Hausdorff space is normal.

Problem 4. Prove that every compact metric space is complete.

Problem 5. Prove that components of an arbitrary topological space are closed.

Problem 6. Suppose X is contractible and Y is path connected. Prove that any two maps from X into a Y are homotopic.

Problem 7. Let f be a continuous bijection of a compact space X onto a Hausdorff space Y. Prove that f is a homeomorphism. (f is a *bijection* if $f(x) \neq f(x')$ for every $x, x' \in X$ such that $x \neq x'$.)

Problem 8. Is there a space X with the component of some point different than its quasi-component? (For a point x in a space X, the *quasi-component* of x in X is the intersection of all closed and open sets containing x.)

Problem 9. Let $\{U_1, U_2, \ldots, U_n\}$ be an open covering of a normal space X. Prove that there are open sets $\{V_1, V_2, \ldots, V_n\}$ covering X so that $\overline{V_i} \subset U_i$ for each $i = 1, \ldots, n$.

Problem 10. Let x_0 and x_1 be two points of a path connected space X. Prove that $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic.

Problem 11. Suppose $p : E \to X$ is a covering map. Let α and β be maps of $[0,1]$ into E such that $p \circ \alpha = p \circ \beta$ and $\alpha(0) = \beta(0)$. Show that $\alpha = \beta$.

Problem 12. Prove that there is a continuous map of the Cantor set onto the Hilbert cube $[0,1]^\infty$.