
1. (11 points) State the definition of a compact set. Prove or disprove the union of two compact sets is compact.

2. (11 points) State the definition of connected space and arc connected space. Show that $S^2 = \{x \in R^3 \mid \|x\| = 1\}$ is connected.

3. (11 points) State the definition of simple connectedness. Prove that the product of two simple connected space is simple connected.

4. (11 points) Let $I = [0, 1]$ Given continuous $f : I \rightarrow I$ prove that the graph of $f = \{(x, f(x)) \in I \times I\}$ is homeomorphic to I .

5. (11 points) Prove that each metric space is a Hausdorff space.

6. (11 points) Find the fundamental group of the following figure in the plane:

7. (11 points) State Brower's fixed point theorem concerning continuous functions from the the unit disc into itself. Explain how does the proof uses the concept of retract and the fundamental group of the circle.

8. (11 points) State Seifert Van-Kampen theorem. Outline how to use a special case (when the intersection of the two spaces is simple connected) to get the fundamental group of "figure 8".

9. (11 points) How is the fundamental group of a knot is defined? What is the fundamental group of an unknotted circle?