Design Theory Prelim
January 9, 2009

1. A STS(19) can be constructed in various ways.
    a. If you were to use the Skolem Construction, write the necessary ingredient(s), then in each of the following cases find the triple containing the given pair of symbols:
        i. ∞ and (3,1)
        ii. (4,3) and (3,1)
    b. If you were to use Wilson's Construction, write the necessary ingredient(s), then in each of the following cases find the triple containing the given pair of symbols:
        i. ∞_1 and 14
        ii. 13 and 14

2. A BIBD(v, k, λ) is a PBD(v, λ) in which each block has size k and each pair of points occurs together in exactly λ blocks.
    a. Show that a necessary condition for the existence of a BIBD(v, 4, 1) is that $v \equiv 1$ or 4 (mod 12).
    b. Find a cyclic BIBD(13, 4, 1) by using difference methods.
    c. Find a BIBD(13, 4, 1) by using 2 MOLS(3).

3. A BIBD(v, k, λ) is said to be nearly resolvable if the set of blocks can be partitioned into sets, called near parallel classes, each of which contains $(v - 1)/k$ vertex-disjoint blocks.
    a. How many near parallel classes would a nearly resolvable BIBD(v, k, λ) contain?
    b. What does this imply about necessary conditions for the existence of a nearly resolvable BIBD(v, 4, λ) when $\lambda \equiv 1$ or 2 (mod 3)?
    c. In view of Questions (2a) and (3b), can a nearly resolvable BIBD(v, 4, 1) exist? Why or why not?
    d. Construct a nearly resolvable BIBD(v, 4, λ) for the smallest possible value of $v > 1$ (you are free to choose the value of λ).

4. Describe 2 different ways to construct 2 MOLS(35). In each case, indicate:
    a. Why the ingredients you use in your constructions are known to exist, and
    b. How you would use these ingredients to make the MOLS(35).

5. Provide necessary and sufficient conditions for the existence of the following latin squares with the following properties, proving your results in each case:
    a. Idempotent and symmetric
    b. Idempotent