Probability Preliminary Exam (2008)

1 (10 points) Independent shocks occur to a system according to a Poisson process of rate 2 per unit time. Suppose effect of a shock on the system lasts an exponential time of mean 0.2, independent of other shocks. Find the probability that the system is feeling the effect of a shock at time 4.

2 (10 points) Let X_n be a discrete time Markov chain on states $0, 1, 2, \ldots$ with 1-step transition probabilities $p_{01} = 1$, $p_{10} = 2/3$, $p_{12} = 1/3$, $p_{21} = 2/3$, $p_{23} = 1/3$, $\ldots$.
(a) Find the long run fraction of steps the Markov chain visits state 0.
(b) From state 10, find the expected number of steps to state 0.

3 (10 points) A workshop has two machines and one repairman. Machine A is up for an exponential time of mean 5 and then breaks down, and machine B is up for an exponential time of mean 4 and then breaks down. The repair time for each machine is exponential of mean 1 and a down machine is repaired immediately when the repairman is free. At the steady state of the system, how long do you expect to wait until both machines are down?

4 (10 points) An item with exponential life time of mean 5 is placed in service. The service will be interrupted if the item is out of order, but a bad item will only be discovered and replaced at an inspection which is performed at equally spaced time interval of length 2. Suppose the replacement time is random of mean 1, and the inspection stops when an item is being replaced and resumes after a new item is installed.
(a) Find the long run fraction of time when the service is interrupted.
(b) In the long run, what is the probability that you will find an item is functioning and will stay functioning for at least 1 unit of time?