On the Number of Perfect Matchings in a Graph

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Abstract: Petersen (1891) showed that every 2-connected cubic graph has at least one perfect matching. Tutte (1947) established a characterization of graphs which possess a perfect matching, and strengthened Petersen's Theorem by showing that every edge in a 2-connected cubic graph is contained in some perfect matching of the graph.

An edge e of a graph G is *admissible* if there is at least one perfect matching of G which contains it. A connected graph of order at least two is *matching covered* if every edge in it is admissible. Our interest here is in counting the number of perfect matchings in a graph. Clearly, in this context, we may restrict our attention to matching covered graphs.

We denote the number of perfect matchings in a graph G by $\Phi(G)$. In this talk I shall present a brief survey of what I know about this function Φ. There are intriguing unsolved problems related to the number of perfect matchings in bipartite matching covered graphs with minimum degree at least three (*On the number of perfect matchings in a bipartite graph* by Carvalho, Lucchesi and Murty; *SIAM J. Disctrete Math, Volume 27 (2013), No. 2, pp 940-958*).