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Effects of Crossed Electric and Magnetic Fields on Spectra of Rydberg Atoms

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ABSTRACT: In lots of studies of laboratory and astrophysical plasmas, there are encountered Rydberg atoms of very large values of the principal quantum number. These atomic states are very sensitive to electric and magnetic fields of the surrounding medium. While interpreting spectra of such excited systems one faces the problem of a huge array of radiative transitions between highly excited atomic levels. Moreover the problem is significantly complicated by external electric and magnetic fields because of the absence of standard selection rules typical for the spherical quantization. The analytical expression in the parabolic representation for dipole matrix elements obtained by Gordon contains hyper-geometric series and has a very complex structure. The coordinate matrix element at the presence of electric and magnetic fields can be calculated in a specific representation which is closely related to the parabolic quantization on two different axes. This matrix element depends in a complex way on the transition probabilities in the parabolic coordinate system (Gordon's formulas) and Wigner d-functions. This circumstance leads to even greater computational difficulties. A method of the simplification of these complicated expressions for transition probabilities is demonstrated. The semiclassical approximation for coordinate matrix elements (Gulayev) and recurrence properties of Wigner d-functions are used. The $H_{n\ß}$ line is under consideration. Specific calculations for the transition 10 - 8 in the case of parallel and perpendicular fields are presented.

1. INTRODUCTION

In lots of studies of laboratory and astrophysical plasmas, there are encountered Rydberg atoms of very large values of the principal quantum number. In these studies, there are two fundamental problems. The first one is connected with the influence of external electric F and magnetic B fields on spectra of Rydberg atoms. This problem is related to the combined Stark-Zeeman eect. It turns out that a suitable description of a hydrogen atom in external F - B fields requires a transition to a special basis associated with taking into account the symmetry properties of the Coulomb field. The second problem is related to the complicated structure of the array of radiative transitions between Rydberg atomic states. So it seems to be a very complicated problem to find a reasonable treatment for the array of spectral lines transition probabilities in the parabolic quantum numbers presentation with respect to the adequate description of the array. In the present work, we show how one can obtain universal formulas for the radiation intensity of a hydrogen-like atom in external electric and magnetic fields.

In order to describe Stark broadening in plasmas it's convenient to use the parabolic representation, instead of the spherical coordinate system. However expressions for dipole matrix elements in this basis obtained by Gordon [1] contain hyper-geometric series, which makes calculations of intensities very cumbersome. Moreover, one faces a huge growth of the transition array for Rydberg atomic states(it grows proportionally to n^4, where n is the principle quantum number). A solution to this problem was given by Gulayev in [2, 3]. He obtained the semiclassical approximation for coordinate matrix elements. The problem of the joint action of crossed electric and magnetic fields on Rydberg atomic states still is not solved properly-in the sense of making it possible to calculate such spectra.

Transition probabilities in the spherical coordinate system have been deeply studied in dierent limits [46]. Also the orbital quantum number l follows the selection rule, which allows one to make fast calculations of dipole matrix

elements. However, the energy shift in a constant electric field has a simple form in the parabolic representation. In the present paper the problem of a large transition array will be solved for highly exited energy levels by establishing approximate selection rules for parabolic quantum numbers. For the first time a hydrogen atom in external electric F and magnetic B fields was considered in the framework of classical mechanics in [7].The quantum treatment was presented in [8]. The symmetry of the Coulomb field can be used to change the representation. The Hamiltonian of the electron in the

Coulomb field and external F-B fields has the following form

$$H = \frac{\mathbf{p}^2}{2} - \frac{Z}{r} + \mathbf{Fr} + \frac{1}{2c}\mathbf{BL} \tag{1}$$

Here p, r and L are the momentum,the coordinate and the angular momentum operators of the electron, respectively, Z is the nuclear charge. This formula (1) and every other in this paper is written in the atomic units. The perturbation $\mathbf{Fr} + \frac{1}{2c}\mathbf{BL}$ can be rewritten in another way.

$$\Delta H = \mathbf{Fr} + \frac{1}{2c}\mathbf{BL} = \mathbf{E}_1\mathbf{J_1} + \mathbf{E}_2\mathbf{J_2} \tag{2}$$

where

$$\mathbf{J_{1,2}} = \frac{1}{2}(\mathbf{L} \pm \mathbf{A}) \tag{3}$$

A is the specific constant of motion in the Coulomb field - the RungeLenz vector.

$$\mathbf{E}_{1,2} = \frac{1}{2c}\mathbf{B} \mp \frac{3}{2}n\mathbf{F} \tag{4}$$

We can do this, because in the Coulomb field there is a relation between the RungeLenz vector and the radius-vector:

$$\mathbf{A} = -\frac{2}{3n}\mathbf{r} \tag{5}$$

The energy shift is equal to

$$\Delta E = E_1 n' + E_2 n'' \tag{6}$$

where n' and n'' are projections of (3) on vectors (4).

The vectors (3) have properties of an angular momentum(see for example [9]). Moreover, projections of (3) on the same direction (z-axis) are related to the parabolic quantum numbers [9]

$$\begin{cases} i_2 - i_1 = n_1 - n_2 \\ i_2 + i_1 = m \end{cases} \tag{7}$$

where $i_{1,2}$ are projections of (3) on z direction(quantization axis), m is the magnetic quantum number.

Using the angular momentum properties of (3) one can change the representation from i_1; i_2 to n', n'' .

$$|n, n', n'' >= \sum_{i_1=-j}^{j} \sum_{i_2=-j}^{j} d^j_{i_1 n'}(\alpha_1) d^j_{i_2 n''}(\alpha_2) |n, i_1, i_2 > \tag{8}$$

where $d^{j}_{m_1 m_2}(\beta)$ is the Wigner d-function.

$$j = \frac{n-1}{2} \tag{9}$$

Here in (8) $\alpha_{1,2}$ are the angles between vectors $J_{1,2}$ and $E_{1,2}$.

$$cos\alpha_{1,2} = \frac{\frac{1}{2c}B \mp \frac{3}{2}nFcos\theta}{E_{1,2}} \tag{10}$$

where θ is the angle between the electric F and magnetic fields B.

Calculation of dipole matrix elements in this representation was presented in [10]. The general expression for coordinate matrix element in the basis of n', n'' has the following form

$$a^{\bar{n}\bar{n}'\bar{n}''}_{nn'n''} = \sum_{\bar{i}_1=-\bar{j}}^{\bar{j}} \sum_{\bar{i}_2=-\bar{j}}^{\bar{j}} \sum_{i_1=-j}^{j} \sum_{i_2=-j}^{j} d^{\bar{j}}_{\bar{i}_1\bar{n}'}(\bar{\alpha}_1) d^{\bar{j}}_{\bar{i}_2\bar{n}''}(\bar{\alpha}_2) d^{j}_{i_1 n'}(\alpha_1) d^{j}_{i_2 n''}(\alpha_2) a^{\bar{n}\bar{i}_1\bar{i}_2}_{n i_1 i_2} \tag{11}$$

where *a* = X, Y, Z. Here n relates to the upper atomic state.

As a result, the number of terms in (11) grows proportionally to n4. However, the usage of Gulayev's results and the specific properties of the Wigner d-functions allows one to make a significant simplification of (11). In the present paper we consider the $H_{n\beta}(\Delta n = n - \bar{n} = 2)$ lines.

2. DERIVATION OF DIPOLE MATRIX ELEMENTS

Our purpose is a simplification of the complicated formula (11). The main problem with this expression is the presence of four sums. The number of terms in this sum is proportional to n^4. The Wigner d-functions also have a complex structure. They can be expressed analytically in terms of the Jacobi polynomials [11].The main idea is to use the combination of the important results from [2, 3] and the d-function properties [11].

In works [2, 3] the authors introduced a new quantum number K

$$K = (n_1 - n_2) - (\bar{n}_1 - \bar{n}_2) \tag{12}$$

The energy shift can be rewritten by using the quantum number K as follows

$$\frac{\Delta E}{\omega_F} = Kn + \Delta nk \tag{13}$$

where $k = \bar{n}_1 - \bar{n}_2$ and $\omega_F = \frac{3}{2}F$

$$Z^m_m = \frac{1}{4}b\left[(\bar{n}_1 + m + 2)(\bar{n}_1 + 2)\delta_{K,+2} + (\bar{n}_2 + m + 2)(\bar{n}_2 + 2)\delta_{K,-2}\right] \tag{14}$$

where $b^2 = \frac{4n\bar{n}}{(n-\bar{n})^2}$. Here m is the absolute value of the magnetic quantum number $\delta_{k,l}$ is the Kronecker delta symbol.

We can split transitions into special series corresponding to specific values of K. Thus, we will obtain two selection rules: the first one for the K-determination of series, the second one is the selection rule for magnetic quantum number m. Proceeding to the i_1, i_2 representation we obtain approximate selection rules for the parabolic quantum numbers.

$$\begin{cases} (i_2 - i_1) - (\bar{i}_2 - \bar{i}_1) = \pm 2 \\ i_2 + i_1 = \bar{i}_2 + \bar{i}_1 \end{cases} \tag{15}$$

The solution of the system (15) has the following form

$$\begin{cases} i_2 = \bar{i}_2 \pm 1 \\ i_1 = \bar{i}_1 \mp 1 \end{cases} \tag{16}$$

The parabolic quantum numbers satisfy the following relation

$$n = n_1 + n_2 + |m| + 1 \tag{17}$$

Using (17) and (7) one can obtain the relation between n_1, n_2 and i_1, i_2

$$\begin{cases} n_1 = \frac{n - |i_1 + i_2| + i_2 - i_1 - 1}{2} \\ n_2 = \frac{n - |i_1 + i_2| + i_1 - i_2 - 1}{2} \end{cases} \tag{18}$$

Then it is necessary to substitute (14), (16) and (18) into (11).

$$Z^{\bar{n}\bar{n}'\bar{n}''}_{1,2nn'n''} = \sum_{\bar{i}_1=-\bar{j}}^{\bar{j}} \sum_{\bar{i}_2=-\bar{j}}^{\bar{j}} d^{\bar{j}}_{\bar{i}_1\bar{n}'}(\alpha_1) d^{\bar{j}}_{\bar{i}_2\bar{n}''}(\alpha_2) d^{j}_{\bar{i}_1\pm 1 n'}(\alpha_1) d^{j}_{\bar{i}_2\mp 1 n''}(\alpha_2) G_{1,2}(\bar{i}_1, \bar{i}_2) \tag{19}$$

where

$$G_1 = (\frac{n}{2} - \bar{i}_1)(\frac{n}{2} + \bar{i}_2)$$

$$G_2 = (\frac{n}{2} + \bar{i}_1)(\frac{n}{2} - \bar{i}_2)$$

After that we have to use recurrence relations for the d-functions [11]

$$d^{j}_{m_1,m_2}(\beta) = \sqrt{\frac{j - m_2}{j - m_1}} cos(\frac{\beta}{2}) d^{j-\frac{1}{2}}_{m_1+\frac{1}{2},m_2+\frac{1}{2}}(\beta) - \sqrt{\frac{j + m_2}{j - m_1}} sin(\frac{\beta}{2}) d^{j-\frac{1}{2}}_{m_1+\frac{1}{2},m_2-\frac{1}{2}}(\beta) \tag{20}$$

$$d^{j}_{m_1,m_2}(\beta) = \sqrt{\frac{j - m_2}{j - m_1}} sin(\frac{\beta}{2}) d^{j-\frac{1}{2}}_{m_1-\frac{1}{2},m_2+\frac{1}{2}}(\beta) + \sqrt{\frac{j + m_2}{j - m_1}} cos(\frac{\beta}{2}) d^{j-\frac{1}{2}}_{m_1-\frac{1}{2},m_2-\frac{1}{2}}(\beta) \tag{21}$$

In order to use (20) and (21) it is necessary to use these relations twice: For $Z^{\bar{n}\bar{n}'\bar{n}''}_{1nn'n''}$ relation (20), for $Z^{\bar{n}\bar{n}'\bar{n}''}_{2nn'n''}$ (21). In the limit $n, \bar{n} \gg 1$ one can notice that factors in the recurrence relations and $G_{1,2}$ coincide. It allows one to use the orthogonality relation for d-functions

$$\sum_{m_3=-j}^{j} (-1)^{m_3 - m_2} d^{j}_{m_2,m_3}(\beta) d^{j}_{m_3,m_1}(\beta) = \delta_{m_1,m_2} \tag{22}$$

After that we can obtain the Z-coordinate matrix element in the representation of states (8)

$$Z_{nn'n''}^{\bar{n}\bar{n}'\bar{n}''} = \frac{1}{4}b(-1)^{\Delta\bar{n}'+\Delta\bar{n}''}\left[Z_{1nn'n''}^{\bar{n}\bar{n}'\bar{n}''} - Z_{2nn'n''}^{\bar{n},\bar{n}',\bar{n}''}\right] \tag{23}$$

$$Z_{1nn'n''}^{\bar{n}\bar{n}'\bar{n}''} = \left[(\frac{n}{2}-n')cos^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'+1} - sin(\alpha_1)\sqrt{(\frac{n}{2}-n')(\frac{n}{2}+n')}\delta_{\bar{n}',n'} + \right.$$
$$\left. +(\frac{n}{2}+n')sin^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'-1}\right] \times \left[(\frac{n}{2}-n'')sin^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''+1} + \right.$$
$$\left. +sin(\alpha_2)\sqrt{(\frac{n}{2}-n'')(\frac{n}{2}+n'')}\delta_{\bar{n}'',n''} + (\frac{n}{2}+n'')cos^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''-1}\right]$$

$$Z_{2nn'n''}^{\bar{n}\bar{n}'\bar{n}''} = \left[(\frac{n}{2}-n'')cos^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''+1} - sin(\alpha_2)\sqrt{(\frac{n}{2}-n'')(\frac{n}{2}+n'')}\delta_{\bar{n}'',n''} + \right.$$
$$\left. +(\frac{n}{2}+n'')sin^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''-1}\right] \times \left[(\frac{n}{2}-n')sin^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'+1} + \right.$$
$$\left. +sin(\alpha_1)\sqrt{(\frac{n}{2}-n')(\frac{n}{2}+n')}\delta_{\bar{n}',n'} + (\frac{n}{2}+n'')cos^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'-1}\right]$$

Derivation of the expression for the X-matrix element is similar to the Z-case. Selection rules for K and for the magnetic quantum number are described by the following system

$$\begin{cases}(i_2-i_1)-(\bar{i}_2-\bar{i}_1)=\pm 1\\ |i_2+i_1|=|\bar{i}_2+\bar{i}_1|\pm 1\end{cases} \tag{24}$$

System (24) leads to four possibilities

$$\begin{cases}\begin{cases}i_2=\bar{i}_2\pm 1\\ i_1=\bar{i}_1\end{cases} & \begin{cases}i_2=\bar{i}_2\\ i_1=\bar{i}_1\pm 1\end{cases}\end{cases} \tag{25}$$

Expressions for the X-matrix elements correspond to K±1

$$X_m^{m-1} = \frac{1}{4}b\left[\sqrt{n_1(n_1+m)(\bar{n}_1+m)(\bar{n}_2+m)}\delta_{K,+1}+\sqrt{n_2(n_2+m)(\bar{n}_2+m)(\bar{n}_2+m)}\delta_{K,-1}\right] \tag{26}$$

$$X_m^{m+1} = \frac{1}{4}b\left[\sqrt{n_1(n_1+m)\bar{n}_1\bar{n}_2}\delta_{K,+1}+\sqrt{n_2(n_2+m)\bar{n}_2\bar{n}_2}\delta_{K,-1}\right] \tag{27}$$

In order to achive the coincidence of mutual factors in the recurrence relations and expressions (26-27) one has to use (20-21), in a special way. For the first case in (25) it is necessary to use relation (20) and after that relation (21). In distinction, for the second case in (25) one has to use (20) after (21). After these manipulations it is easy to obtain the X-matrix element

$$X_{nn'n''}^{\bar{n}\bar{n}'\bar{n}''} = \frac{1}{4}b(-1)^{\Delta\bar{n}'+\Delta\bar{n}''}\left[X_{1nn'n''}^{\bar{n}\bar{n}'\bar{n}''} - X_{2nn'n''}^{\bar{n},\bar{n}',\bar{n}''} - X_{3nn'n''}^{\bar{n}\bar{n}'\bar{n}''} + X_{4nn'n''}^{\bar{n}\bar{n}'\bar{n}''}\right] \tag{28}$$

$$X^{\bar{n}\bar{n}'\bar{n}''}_{1nn'n''} = \left[(\frac{n}{2}-n')cos^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'+1} - sin(\alpha_1)\sqrt{(\frac{n}{2}-n')(\frac{n}{2}+n')}\delta_{\bar{n}',n'} + \right.$$
$$\left. +(\frac{n}{2}+n')sin^2\left(\frac{\alpha_1}{2}\right)\delta_{\bar{n}',n'-1}\right] \times \left[\frac{1}{2}sin(\alpha_2)\left((\frac{n}{2}-n'')\delta_{\bar{n}'',n''+1} - (\frac{n}{2}+n'')\delta_{\bar{n}'',n''-1}\right) + \right.$$
$$\left. +\delta_{\bar{n}'',n''}cos(\alpha_2)\sqrt{(\frac{n}{2}+n'')(\frac{n}{2}-n'')}\right]$$

$$X^{\bar{n}\bar{n}'\bar{n}''}_{2nn'n''} = \left[(\frac{n}{2}-n'')sin^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''+1} + sin(\alpha_2)\sqrt{(\frac{n}{2}-n'')(\frac{n}{2}+n'')}\delta_{\bar{n}'',n''} + \right.$$
$$\left. +(\frac{n}{2}+n'')cos^2\left(\frac{\alpha_2}{2}\right)\delta_{\bar{n}'',n''-1}\right] \times \left[\frac{1}{2}sin(\alpha_1)\left((\frac{n}{2}-n')\delta_{\bar{n}',n'+1} - (\frac{n}{2}+n')\delta_{\bar{n}',n'-1}\right) + \right.$$
$$\left. +\delta_{\bar{n}',n'}cos(\alpha_1)\sqrt{(\frac{n}{2}+n')(\frac{n}{2}-n')}\right]$$

Here $X^{\bar{n}\bar{n}'\bar{n}''}_{3nn'n''}$ can be obtained by switching $n' \Leftrightarrow n''$(for bar values too) and $\alpha_1 \Leftrightarrow \alpha_2$ in $X^{\bar{n}\bar{n}'\bar{n}''}_{1nn'n''}$. The same connection exists between $X^{\bar{n}\bar{n}'\bar{n}''}_{2nn'n''}$ and $X^{\bar{n}\bar{n}'\bar{n}''}_{4nn'n''}$.

Using the result for the X-dipole matrix element one can obtain the expression for the Y-matrix element. The hydrogen wave function is proportional to $e^{im\varphi}$, $X \sim cos\varphi$,$Y \sim sin\varphi$. Using well-known relations $cos(z) = \frac{e^{iz}+e^{-iz}}{2}$ and $sin(z) = \frac{e^{iz}+e^{-iz}}{2i}$ one can obtain

$$Y^{\bar{n}\bar{n}'\bar{n}''}_{nn'n''} = \frac{1}{4i}b(-1)^{\Delta\bar{n}'+\Delta\bar{n}''}\left[X^{\bar{n}\bar{n}'\bar{n}''}_{1nn'n''} - X^{\bar{n},\bar{n}',\bar{n}''}_{2nn'n''} - X^{\bar{n}\bar{n}'\bar{n}''}_{3nn'n''} - X^{\bar{n}\bar{n}'\bar{n}''}_{4nn'n''}\right] \tag{29}$$

3. RESULTS

In order to analyze obtained results we consider the ratio of Zeeman and Stark shifts denoted below as u. The intensity is proportional to the square of the absolute value of coordinate matrix element in the dipole approximation.

$$u = \frac{B}{3cnE} \tag{30}$$

The reduced energy shift is equal to

$$\omega = (\bar{E}_1\bar{n}' + \bar{E}_2\bar{n}'' - E_1n' - E_2n'')/\Omega_{FB} \tag{31}$$

$$\Omega_{FB} = \frac{1}{2c}B + \frac{3}{2}nF \tag{32}$$

Calculations of the intensity in the case of parallel fields are presented in figure 1. In fig.1(a) one can see pure Stark effect. This result is in agreement with [3]. The absence of the central component is typical for the lines $H_{n\beta}$ without a magnetic field. In the presence of the magnetic field B one can observe how intensity components merge together (fig.1 b,c). Finally, the when Zeeman shift becomes much larger than the Stark shift we obtain the picture of the pure PaschenBack effect.

Expressions (14),(26),(27) contain Kronecker delta symbols. This circumstance leads to the fact that for highly excited levels the numbers n', n'' follow the selection rules.

In the absence of a magnetic field: $\alpha_1 = \pi$ and $\alpha_2 = 0$ (see (10)). If one substitute these values of the angles in expressions (23), (28,29) and change $n' \Rightarrow -n'$ one would retrieve the usual Stark eect and formulas (14),(26),(27). In the opposite limit of the large Zeeman shift $\alpha_{1,2}$ = 0 and because of the selection rules for n', n'' expressions (23), (28,29) one would

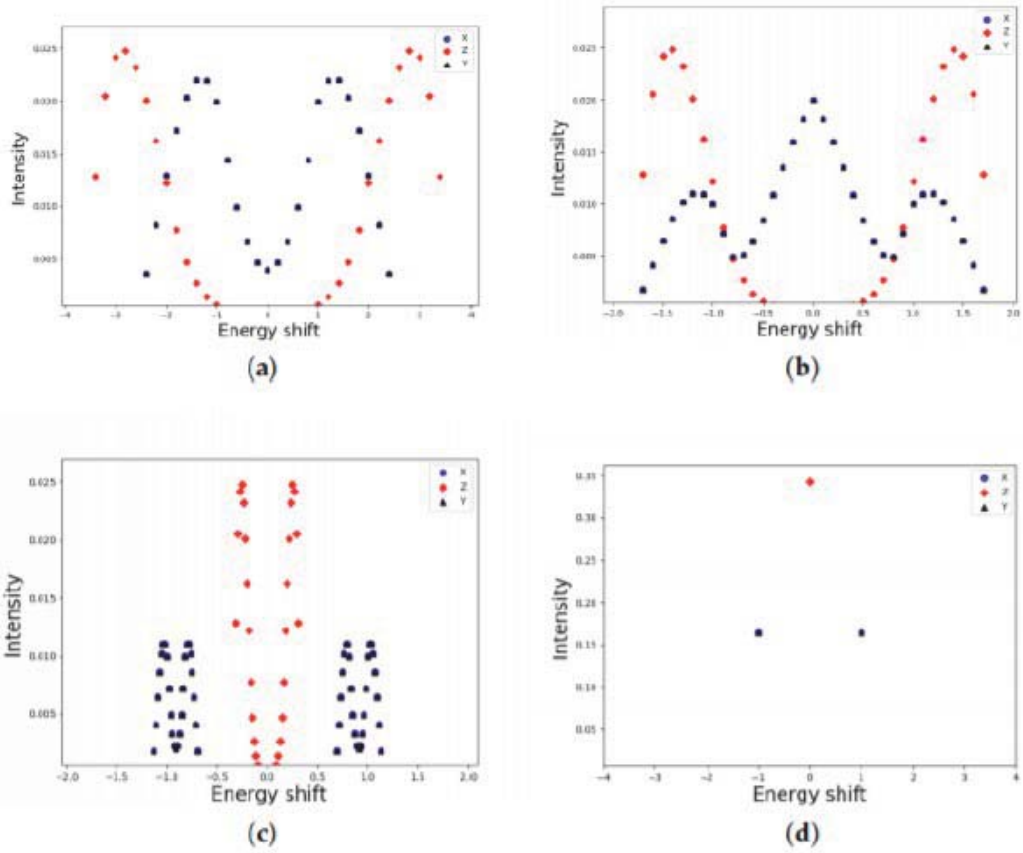

Figure 1: Transition from n=10 to n=8.Intensity(divided by the sum of intensities of all components) as the function of the reduced energy(31-32) in the case of parallel fields (θ = 0): a) u=0 , b) u=1 c) u=10 d) u=1000; $u = \frac{B}{3cnE}$.

reproduce the Paschen Back eect. Due to the symmetry of the system, the intensity components of the radiation polarized in the X and Y directions coincide.

Figure 2a presents the case of zero magnetic field and the electric field parallel to the x-axes. It is seen that the intensity profiles, corresponding to the X- and Z-polarizations, got interchanged. Due to the decrease in the degree of the symmetry, the matrix elements begin to appear in pairs of mismatched intensity components (fig.2b). In comparison with the case of parallel fields, the transition to the Paschen Back eect occurs already at u = 10 (fig.3c).

The non-conservation of the full integrated intensity is related to the fact

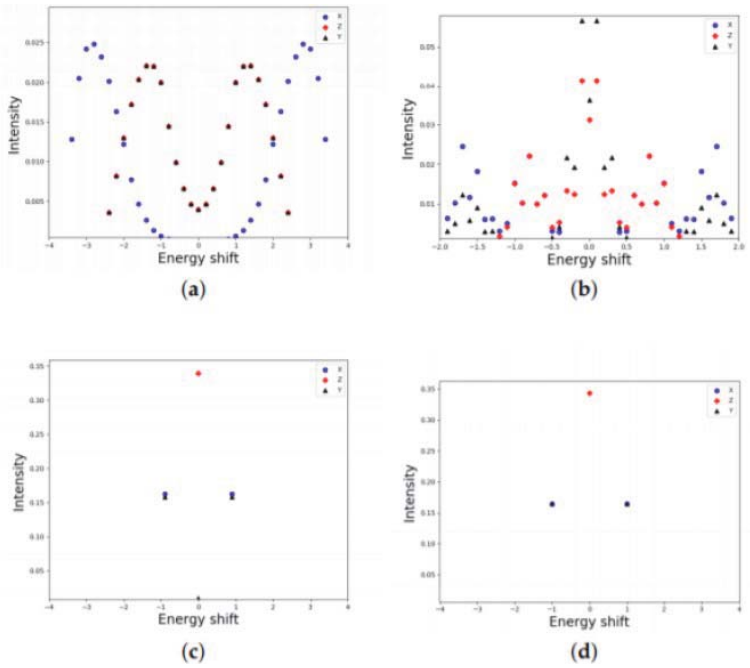

Figure 2: The same as in fig.1 but for perpendicular fields ($\theta = \frac{\pi}{2}$)

that all three types of matrix elements(polarizations) are calculated instead of two.

4. CONCLUSION

Studies of astrophysical and laboratory plasmas frequently employ analysis of hydrogen spectral lines. While dealing with highly exited(Rydberg) atomic states one faces two fundamental problems. The first one is related to the complex structure of accurate expressions for dipole-matrix elements in the parabolic representation obtained by Gordon [1,6]. The second one is the in"uence of magnetic and electric fields on spectra of a hydrogen-like atom. Gulayev in his works on astrophysical plasma spectroscopy [2, 3] obtained the semiclassical approximation for dipole-matrix elements. Moreover, he pointed out that the intensity of the radiation is strongly depends on the quantum number (12). Because of this circumstance one can neglect a large part of radiation transitions. The quantum consideration of a hydrogen-like atom in external F-B [8] gave a signi"cant impetus to solving this problem. In work [10] the authors used these results to calculate dipole matrix elements in the representation of states from (8). Practically, while dealing with large principal quantum numbers one faces again complicated sums (11) with n4 terms. It was shown in the present paper how to solve the problem. Using the semiclassical approximation for dipole matrix elements and the properties of d-functions we reduced the complicated formula (11), which contains the complex hyper geometric series in $a^{\bar{n}\bar{i}_1\bar{i}_2}_{ni_1i_2}$ and the Jacobi polynomials in the d-functions, to expressions (23) and (28)-(29). These formulas contain trigonometric functions and Kroneker-delta symbols, which expresses a new selection rules for the quantum numbers n0; n00. Moreover, we emphasize the universality of these formulas. The new semiclassical expressions describe any transition with Δn = 2.

In figures 1 and 2 we presented specific calculations related to the transition 10-8. We considered the cases of parallel and perpendicular fields.Using Figures 1 and 2, it is possible to trace the smooth transition from the pure Stark eect to the Zeeman components.By gradually increasing the magnitude of the magnetic field, one can observe how intensity components merge together.

In summary , we derived the semiclassical approximation for dipole matrix elements, using Gulaev's formulas and the recurrence relations for the Wigner d-functions. These expressions have universal properties. The initial expression (11) contains n4 terms, the d-functions and the complicated Gordon's([1, 6]) formulas. However, the

simultaneous use of both the semiclassical results and the properties of d-functions leads to the simple, universal formulas (23) and (28)-(29).

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