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REVIEW ARTICLE

Laser-Aided Diagnostics of Oscillatory Electric Fields in Plasmas Based on the Concept of Quasienergy States

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ABSTRACT: Methods for the laser-assisted diagnostics of oscillatory electric fields (OEFs) in plasmas are reviewed. It is shown that several phenomena can be used for this purpose. First, a tunable laser radiation can scan both the structure of energy levels and spectral line profiles of atoms interacting with the OEFs. Second, the laser radiation can cause a saturation of the atomic transition, including the cases where the saturation parameter depends on characteristics of the OEFs. Third, the laser radiation itself can modify both the structure of energy levels and spectral line profiles in such a way that the modification would depend, in particular, on parameters of the OEFs.

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1. INTRODUCTION

Developing methods for diagnosing oscillatory electric fields (OEFs) $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$ in plasmas is of a great practical importance: the OEFs control many physical processes in plasmas. The term OEF embraces both the intrinsic eigen-modes of plasmas (plasma turbulence) and extrinsic fields penetrating into plasmas from the outside. To the latter type belongs, e.g., the field of the microwave radiation used for heating in some magnetic fusion machines.

Currently for diagnosing OEFs in plasmas, there are widely employed methods based on the emission spectroscopy [1]. As an example, we note paper [2], where OEFs were revealed and studied in the peripheral plasma of the tokamak T-10 by using profiles of spontaneously emitted spectral lines of deuterium.

Bringing into play a laser radiation can significantly broaden the scope of spectroscopic diagnostics of OEFs in plasmas. The advantage is the possibility of measuring OEFs with a high temporal and spatial resolutions. In the laser-assisted diagnostics of the OEF $\vec{E}_M(t)$, atoms in a plasma interact with both the laser field $\vec{E}_L(t) = \vec{E}_{0L}\cos\omega_L t$ and with the OEF $\vec{E}_M(t)$. We assume that $\omega_M << \omega_L$ and that the frequency ω_L is close to the frequency of the atomic transition between the upper and lower energy levels.

In magnetic fusion machines, there are various ways for atoms to serve as sensitive "probes" with respect to the OEFs. First, atoms of the main filling gases (hydrogen, deuterium) can be used for diagnosing OEFs in the peripheral plasma. Second, for measuring OEFs in the core plasma one can inject neutral beams (hydrogen, deuterium, helium, litium). Besides, one can also inject pellets (containing, e.g., lithium). As an example, Fig. 1 shows a possible geometry for the laser-assisted diagnostics of OEFs in a tokamak, where atoms of the injected neutral beam play the role of the "probes". In this case, in addition to the OEF, one should also take into account the Lorentz electric field $\vec{E}_b = \vec{V}_b \times \vec{B}/c$, where $\vec{V}_b$ is beam velocity and $\vec{B}$ is the magnetic field.

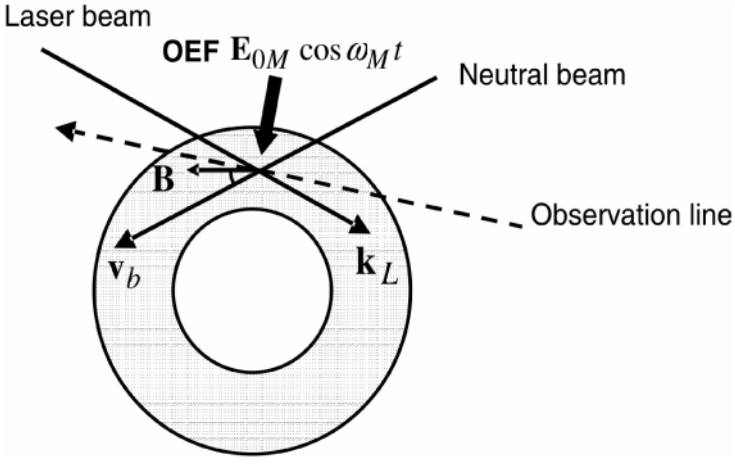

Figure 1: Geometry of the observations for laser-assisted diagnostics of oscillatory electric fields in tokamak plasmas

In this paper we consider physical principles that can serve as the basis for laser-assisted spectroscopic methods for diagnosing OEFs in magnetic fusion plasmas.

2. ABSORPTION SPECTRA OF ATOMS NOT HAVING PERMANENT DIPOLE MOMENTS

The interaction of an atom with the OEF $\vec{E}_M(t)$ can be characterized by the parameter

$$R = \left| d_{\alpha'\alpha''} E_{0M} / [2\hbar(\omega_{\alpha'\alpha''} \pm \omega_M)] \right| \tag{1}$$

where $d_{\alpha'\alpha''}$ is the matrix element of the dipole moment between the closely spaced levels α', α'', and $\omega_{\alpha'\alpha''}$ is the separation between these levels. If R<< 1 (the case of a weak field $\vec{E}_M(t)$), the field $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$ can be measured using the fact that the excitation rate for the two-quantum process, involving one quantum $\hbar\omega_L$ of the laser field $\vec{E}_L(t) = \vec{E}_{0L}\cos\omega_L t$ and one quantum $\hbar\omega_L$ of the field $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$, is proportional to E_{0M}^2. Let us consider, for instance, the excitation scheme shown in Fig. 2, where $\omega_{23} << \omega_{21}$. We assume that in the electric dipole approximation, the single-photon transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ are allowed, whereas the single-photon transition $1 \leftrightarrow 2$ is forbidden.

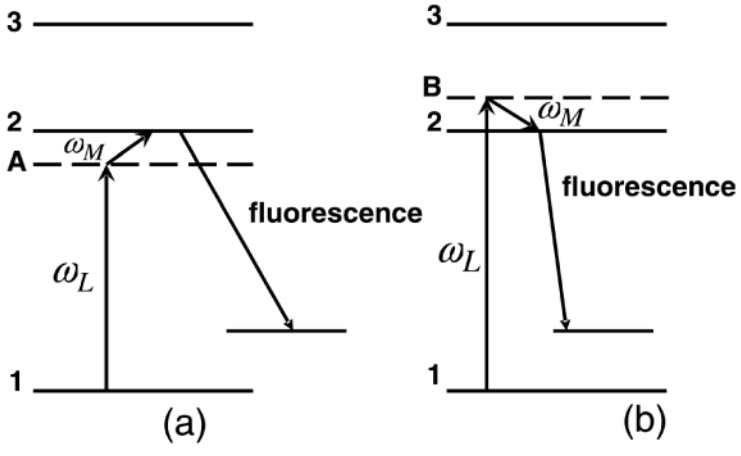

Figure 2: Partial scheme of energy levels of an atom. (a) The excitation of the transition 1→2 Due to the absorption of a laser photon $\hbar\omega_L$ and of an OEF quantum $\hbar\omega_M$. (b) The excitation of the transition 1 → 2 due to the absorption of a laser photon $\hbar\omega_L$ and the emission of an OEF quantum $\hbar\omega_M$

Using the quantum-mechanical perturbation theory (we assume that $R<<1$ for the levels 2 and 3), one can obtain the excitation rates for the two-quantum transitions shown in Fig. 1:

$$W_{1\to 2}^{(+)} = \{E_{0L}^2 E_{0M}^2 d_{13}^2 d_{32}^2 / [8\hbar^2(\omega_{32}+\omega_M)^2]\} L(\omega_{21}-\omega_L-\omega_M,\gamma),$$
$$W_{1\to 2}^{(-)} = \{E_{0L}^2 E_{0M}^2 d_{13}^2 d_{32}^2 / [8\hbar^2(\omega_{32}-\omega_M)^2]\} L(\omega_{21}-\omega_L+\omega_M,\gamma). \quad (2)$$

In Eqs. (2), $L(\omega,\gamma) \equiv (\gamma/2)/(\omega^2+\gamma^2/4)$, γ is the width (FWHM) of the line. The value of $W_{1\to 2}^{(+)}$ corresponds to the process shown in Fig. 2(a), whereas the value of $W_{1\to 2}^{(-)}$ corresponds to the process shown in Fig. 2(b). The excitation rate for the direct single-photon transition 1→3 induced by the laser field is as follows:

$$W_{1\to 3} = [d_{13}^2 E_{0L}^2/(2\hbar^2)] L(\omega_{21}-\omega_L,\gamma). \quad (3)$$

Figure 3 shows the absorption spectrum consisting of the lines $W_{1\to 2}^{(\pm)}$ and $W_{1\to 3}$ for the transitions 1→(2,3). One can see that both the ratios $W_{1\to 2}^{(+)}/W_{1\to 3}$ and $W_{1\to 2}^{(-)}/W_{1\to 3}$ are proportional to E_{0M}^2. Therefore, the ratios $W_{1\to 2}^{(+)}/W_{1\to 3}$ and $W_{1\to 2}^{(-)}/W_{1\to 3}$ can be used to measure E_{OM} by scanning the laser frequency around the frequencies ω_{21} and ω_{31} and observing the fluorescence signals at transitions originated from the levels 2 and 3.

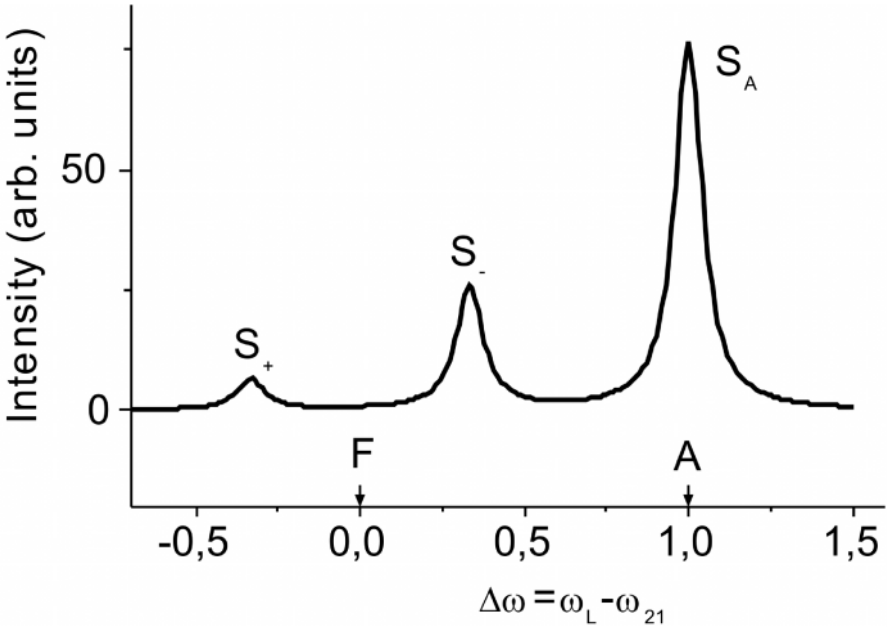

Figure 3: Absorption spectrum for the transitions 1→(2,3) shown in Fig. 2. $S_A \equiv W_{1\to 3}$ is the intensity of the allowed line 1→3; $S_+ \equiv W_{1\to 2}^{(+)}$ is the intensity of the line corresponding to the excitation scheme (a); $S_- \equiv W_{1\to 2}^{(-)}$ is the intensity of the line corresponding to the excitation scheme (b) (cf. Fig. 2). The separation between the lines S_- and S_+ is equal to $2\omega_M$

In the case where the parameter R in Eq. (1) is of the order or greater than unity, the formalism of quasienergy states (QES) is more appropriate for treating the interaction of the atom with the OEF $\vec{E}_M(t)$. According to the Floquet theory (see, e.g., Ref. [3]), for the atom interacting with the OEF $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$, among the wave functions (WFs) of the atom, there exists a basis of the WFs of QES

$$\Phi_n(\vec{r},t) = \exp(-i\varepsilon_n t/\hbar)\sum_k \exp(-ik\omega_M t)\phi_{nk}(\vec{r}) \quad (4)$$

where ε_n is the quasienergy of the level n, and the functions $\phi_{nk}(\vec{r})$ are time independent. We note that the term "quasienergy states" was introduced in Refs. [4,5]. The quasienergies ε_n and the functions $\phi_{nk}(\vec{r})$ can be obtained either numerically, or analytically by solving the Schrödinger equation. Generally they depend on the parameters of the field $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$. Taking into account Eq. (4), one can see that the excitation of the transition $b \to a$ can occur when the following resonance condition is satisfied

$$\omega_L \approx (\varepsilon_a - \varepsilon_b)/\hbar + k\omega_M, \quad k = 0, \pm 1, \pm 2, \ldots$$

where ε_a and ε_b are the quasienergies of the levels a and b, respectively. Using Eq. (4), we find that for determining the excitation rate for the transition $b \to a$, we can use the formula (3), in which the dipole matrix element d_{13} should be replaced by the effective dipole matrix element $D_{ba}^{(k)} = \langle \varphi_b | d | \phi_{ak} \rangle$, where φ_b is the WF of the lower level b. (We assume here that strong the OEF $\vec{E}_M(t)$ perturbs only the upper state a.) As a result, the excitation rate for the transition $b \to a$ can be expressed as

$$W_{b\to a} = [|< \varphi_b | d | \phi_{ak} >|^2 E_{0L}^2/(2\hbar^2)]L((\varepsilon_a - \varepsilon_b)/\hbar - \omega_L + k\omega_M, \gamma).$$

Let us assume that the levels b and a coincide, respectively, with the levels 1 and 2 shown in Fig. 2. In the case of a weak OEF $\vec{E}_M(t)$, according to Eq. (2) the ratio $W_{1\to 2}^{(+)}/W_{1\to 2}^{(-)}$ does not depend on E_{OM}. However, for the strong OEF $\vec{E}_M(t)$ this ratio is a function of the parameters of the OEF. Figure 4 shows, as an example, the ratio $W_{1\to 2}^{(+)}/W_{1\to 2}^{(-)}$ versus the amplitude of the OEF $\vec{E}_M(t)$, calculated for two two-quantum transitions in helium atoms: the transitions $2^1P \to 4^1F$ and $2^3P \to 4^3F$. For helium atoms, the levels $2^{2S+1}P$, $4^{2S+1}F$, and $4^{2S+1}D$ (S = 0; l) play the role of the levels 1, 2, and 3 shown in Fig. 2. Using the dependence of $W_{1\to 2}^{(+)}/W_{1\to 2}^{(-)}$ on the parameters of the OEF, one can measure these parameters in a plasma by observing the laser-induced fluorescence (LIF) signal at the transition from the level 2 to some lower level q. The advantage of this method is that both the fluorescent lines originate from the same level 2 - therefore, a possible collisional transfer of the population from the level 2 to nearby levels does not interfere with the measurements of the OEF.

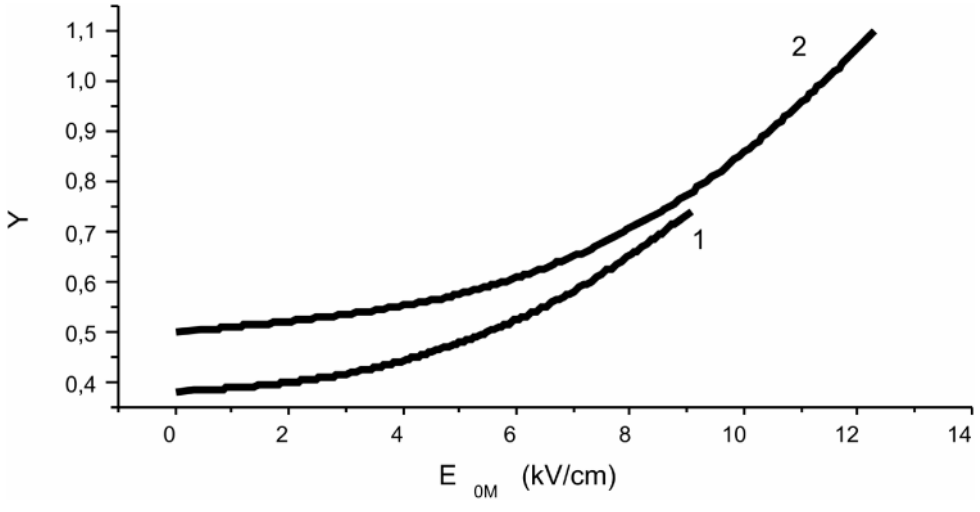

Figure 4: The ratio $Y = W_{1\to 2}^{(+)}/W_{1\to 2}^{(-)}$ versus the amplitude E_{OM} of the OEF for two two-quantum transitions in helium atoms: $2^1P \to 4^1F$ (curve 1) and $2^3P \to 4^3F$ (curve 2). The OEF frequency in this example ω_M was 2.4×10^{11} s^{-1}

3. ABSORPTION SPECTRA OF ATOMS POSSESSING PERMANENT DIPOLE MOMENTS

Let us consider laser-induced transitions in atoms possessing permanent dipole moments ($d_{aa} \equiv <\varphi_a \mid d \mid \varphi_a> \neq 0$). Hydrogen atoms are one of the examples of such atoms. In the case where $d_{aa} \neq 0$, the WF of a QES of the atom, which is in the state *a* and interacts with the OEF $\vec{E}_M(t) = \vec{E}_{0M}\cos\omega_M t$, has the form [6]

$$\Psi_a(\vec{r},t) = \exp[-id_{aa}E_{0M}\sin\omega_M t/(\hbar\omega_M)]\varphi_a(\vec{r}) \tag{5}$$

Let the frequency ω_L of the laser radiation be close to the frequency of the transition $b \leftrightarrow a$. Performing the Fourier series expansion in Eq. (5) and taking into account the formula $\exp(-ix\sin\omega_M t) = \sum_{n=-\infty}^{+\infty} J_n(x)\exp(-in\omega_M t)$, where $J_n(x)$ are the Bessel functions, we find that the effective matrix element of the dipole moment between the levels *a* and *b* has the form: $D_{ab}^{(k)} = J_k(\Delta\beta_{ab})d_{ab}$, where $\Delta\beta_{ab} \equiv (d_{aa} - d_{bb})E_{0M}/(\hbar\omega_M)$. Using the above expression for $D_{ab}^{(k)}$, we obtain the increase of the population ΔN_a of the upper level *a*

$$\begin{aligned}
&\Delta N_a \equiv N_a - N_{a0} = 2^{-1}G_k(N_{b0} - N_{a0})/[1 + (\omega_{ab} + k\omega_M - \omega_L - \sigma_k)^2\tau_{12}^2 + G_k],\\
&G_k \equiv 4W_{ab}^2 J_k^2(\Delta\beta_{ab})\tau_{12}\Gamma^{-1},\ \sigma_k = 2\omega_M^{-1}W_{ab}^2 g_k(\Delta\beta_{ba}),\\
&g_k(\nu) = \sum_{r=1}^{\infty}[J_{k-r}^2(\nu) - J_{k+r}^2(\nu)]/r,\ W_{ab} \equiv d_{ab}E_{0L}/(2\hbar),\ k = 0,\pm1,\pm2,...
\end{aligned} \tag{6}$$

Here τ_{12} and Γ^{-1} are transverse and longitudinal relaxation times, respectively. The quantity τ_{12} is inversely proportional to the impact width of the spectral line $a \rightarrow b$; the quantity Γ controls the relaxation rate of the levels *a* and *b*.

In the case of a weak laser field the expression for ΔN_a in (6) simplifies to:

$$\Delta N_a = \frac{d_{ab}^2 E_{0L}^2}{2\hbar^2}\frac{J_k^2(\Delta\beta_{ab})\tau_{12}\Gamma^{-1}(N_{b0} - N_{a0})}{1 + (\omega_{ab} + k\omega_M - \omega_L)^2\tau_{12}^2},\ k = 0,\pm1,\pm2,... \tag{7}$$

In fact, Eq. (7) determines Blokhinzew's spectrum [6], where the satellite at the frequency $\omega_L = \omega_{ab} + k\omega_M$ has the intensity proportional to $J_k^2(\Delta\beta_{ab})$.

From Eq. (6) it follows that that in the general case the spectrum of the atomic absorption of the laser radiation consists of a set of satellites at the frequencies

$$\omega_L = \omega_{ab} + k\omega_M - \sigma_k,\ k = 0, \pm 1, \pm 2,..., \tag{8}$$

the half-width of the k-th satellite being $\Delta\omega_{k,1/2} = 2(1 + G_k)^{1/2}/\tau_{12}$. We point out that the term σ_k in the denominator of the expression for ΔN_a (see Eq. (6)) is related to the dynamic Stark shift of the quasienergy levels. It depends on both the laser field, and the OEF $\vec{E}_M(t)$. We note that the expression for σ_k was first obtained in Refs. [7, 8]. Figure 5 shows, as an example, the absorption spectrum of the laser radiation by atoms possessing the permanent dipole moments in the states *a* and *b*. It consists of the unshifted line S_0 and of the satellite lines $S_{\pm k}$ (k = 1,2,..). The intensity of these lines, according to Eq. (6), is the function of the amplitude E_{0M} of the OEF. For Fig. 5, the matrix elements of the dipole moment were assumed to be as follows: $d_{ab} = 2.15ea_0$, $d_{aa} - d_{bb} = 3.5ea_0$ where a_0 is the Bohr radius. Profiles shown in Fig. 5 were calculated at E_{0L} = 15 kV/cm, E_{0M} = 7 kV/cm, $\omega_M = 2.4\times10^{11}$ s^{-1}. For the profile (a), the separation between the satellites S_{+k} and S_{-k} is equal to $2k\omega_M$. One can see from Fig. 5 that the allowance for the dynamic Stark shifts σ_k can be important for calculations of the absorption spectra of atoms.

Equations (6) can be used for measuring the amplitude of the microwave field E_{0M} in plasmas. For this purpose one should record the wavelength-integrated intensity of LIF $I_f \propto \Delta N_2$ during the transition from the upper level *a* to one of the lower levels versus the laser field intensity $I_L \propto E_{0L}^2$: $I_f^{-1} \propto \left(1 + I_{k,satur} I_L^{-1}\right)$, $I_{k,satur} I_L^{-1} \equiv G_k^{-1}(E_{0M})$. Here *k* is the number of microwave quanta involved in the resonance (see Eq. (8)). The amplitude E_{0M} can be measured in two ways. In the first way, one could measure the ratio of slopes of experimental dependences $I_f^{-1}(I_L^{-1})$ at two different values of *k* (*k* = *k*′, and *k* = *k*″), and use the dependence of the ratio $G_{k'} / G_{k''}$ on E_{0M}. In the second way, one could use the dependence of the dynamic Stark shift σ_k on the parameters of the OEF $\vec{E}_M(t)$. By scanning the laser frequency around the frequency ω_{ab} and by recording peaks of the LIF intensity, one could tune to multi-quantum resonances (8) corresponding to two different indices $k = k_1$, and $k = k_2$. The laser frequency should be kept constant. In this situation one would have:

$$\omega_{L,1} = \omega_{ab} + k_1 \omega_M - \sigma_{k_1}, \quad \omega_{L,2} = \omega_{ab} + k_2 \omega_M - \sigma_{k_2}. \tag{9}$$

Since ω_M is known, from Eqs. (9) one would determine the dynamic Stark shifts σ_{k_1} and σ_{k_2}. Then, using the fact that the ratio $\sigma_{k_2} / \sigma_{k_1}$ depends on E_{0M} but does not depend on E_{0L}, one could finally find the OEF amplitude E_{0M}.

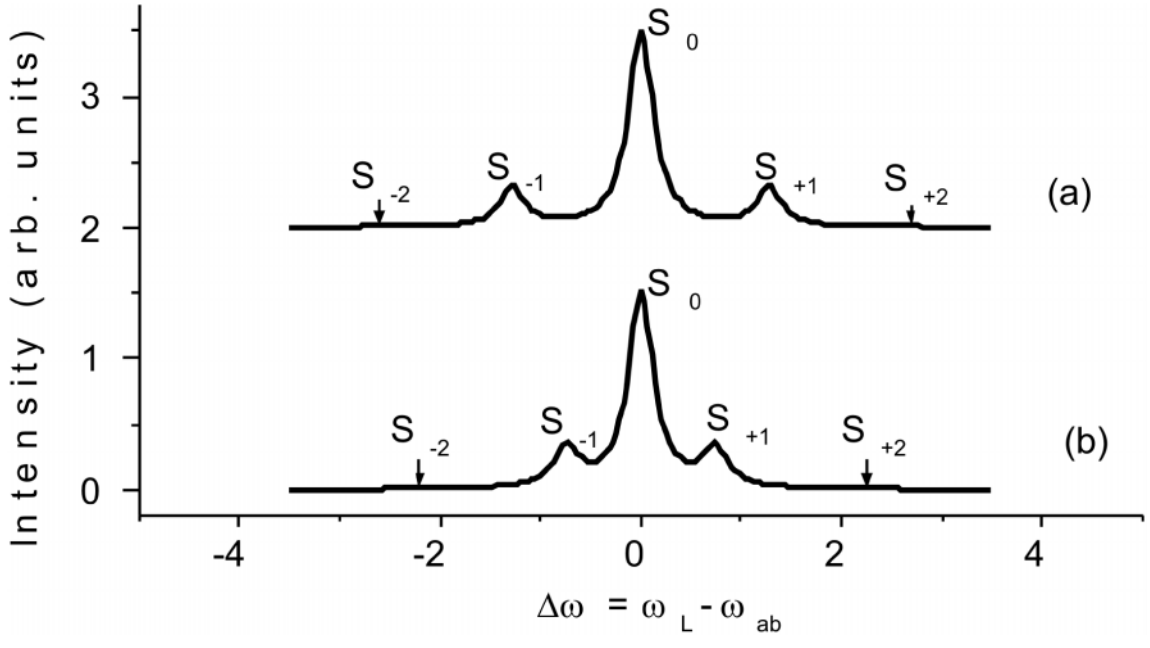

Figure 5: Absorption spectra of the laser radiation in the presence of the field $E_M(t) = E_{0M} \cos \omega_M t$ by atoms possessing permanent dipole moments in their states *a* and *b*. Profile (a) was calculated without the allowance for the dynamic stark shifts (DSS) σ_k of the satellites, whereas profile (b) was calculated with the allowance for the DSS σ_k of the satellites

Now let us consider a situation where a hydrogen (or deuterium) beam travels through a magnetic fusion plasma containing OEF $\vec{E}_M(t)$. In this case, hydrogen atoms are subjected to the combined electric field $\vec{\varepsilon}(t) = \vec{E}_b + \vec{E}_M(t)$, where $\vec{E}_b = \vec{V}_b \times \vec{B} / c$ is the Lorentz electric field. The field $\vec{E}_b(t)$ causes the static Stark splitting of the hydrogen energy level of the principal quantum number n. By a proper choice of the beam velocity, it is possible to realize the situation where this Stark splitting (in the frequency scale) coincides with the frequency ω_M of the OEF $\vec{E}_M(t)$, thus resulting in a resonance. The resonance condition has the form:

$$\omega_M = \frac{3\hbar}{2me} n E_b \,. \tag{10}$$

Under the condition (10), the probability, that the hydrogen atom is in a Stark sublevel of the energy level n, oscillates at the Rabi frequency, which is proportional to the OEF amplitude. This causes an additional splitting of those hydrogen spectral lines, for which the energy level n serves as the upper level or as the lower level. This additional splitting is linear with respect to the OEF amplitude.

As an example, Fig. 6 shows the spectrum of the hydrogen line L_α under the resonance (10) for the upper level $n = 2$. The additional splitting, being linear with respect to the OEF amplitude, can be used as sensitive tool for measuring the OEF amplitude in magnetic fusion plasmas. For tuning into the resonance (10), one should vary the beam energy. The additional splitting would manifest in the spectrum of the fluorescence excited by the laser tuned to the atomic transition, for which the level n (where n satisfies Eq. (10)) serves as the upper level or as the lower level.

For an estimate, let us assume the OEF frequency $\omega_M = 8.7 \times 10^{11} s^{-1}$, the magnetic field $B = 1T$, the hydrogen beam velocity $\vec{V}_b$ being orthogonal to the magnetic field $\vec{B}$. For fulfilling the resonance (10) for the energy level $n = 3$ (which is the upper level of the H_α line), we find that the energy of the hydrogen beam should be 30 keV.

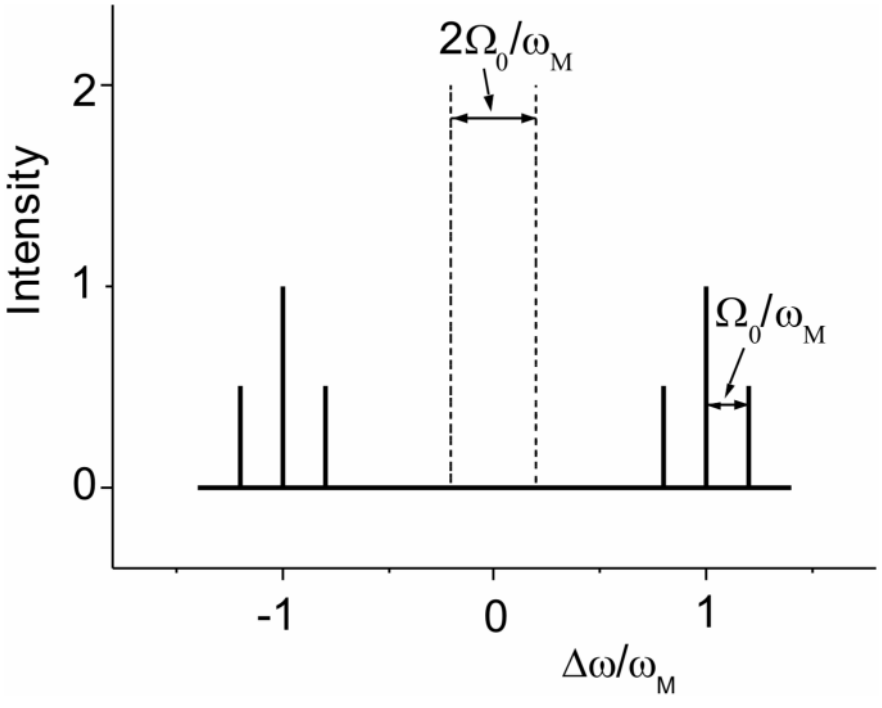

Figure 6: Spectrum of the hydrogen line L_α in two orthogonal electric fields: A lorentz field $\vec{E}_b$ and an oscillatory field $\vec{E}_M(t) = \vec{E}_{0M} \cos\omega_M t$ - under the resonance (10) for the level $n = 2$. The observation is perpendicular to both $\vec{E}_b$ and $\vec{E}_M(t)$. The spectrum was calculated for two orientations of the linear polarization analyzer: parallel to $\vec{E}_b$ (solid line) and parallel to $\vec{E}_M(t)$ (dashed line). The rabi frequency controlling the splitting is $\Omega_0 = \frac{3\hbar}{2me} E_{0M}$

4. CONCLUSIONS

In this review we presented the main physical principles that can serve as the basis for laser-assisted spectroscopic methods for diagnosing OEFs in plasmas. As atomic "probes", one can use either atoms possessing permanent dipole moments (e.g., hydrogen or deuterium atoms) or atoms that do not possess permanent dipole moments (e.g., helium or lithium atoms).

We showed that several phenomena can be used for laser-assisted diagnostics of OEFs in magnetic fusion plasmas. First, a tunable laser radiation can scan spectral line profiles and/or the structure of energy levels of atoms interacting with OEFs. In this case, parameters of the OEFs can be deduced from the spectrum of fluorescence, resulting from the atomic transition from an excited state to a lower state. For this case we assumed that the laser intensity is relatively weak, so that it does not modify the structure of the atomic energy levels.

Second, the laser radiation can cause the saturation of the atomic transition. In this case for some atomic transitions the saturation parameter depends both on the laser intensity and on the parameters of the OEF. The latter can be used for diagnosing the OEF. For this case one needs a sufficiently intense laser radiation that can cause the saturastion.

Third, a combined effect of the OEF and of a strong laser field can modify both the structure of energy levels and spectral line profiles. In this case, the character of the modification is controlled by the quantity, in which parameters of both the OEF and the laser field are represented in an entangled way.

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