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Classical Description of Circular States of a Molecule μpe

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ABSTRACT: We studied the existence of a muonic negative hydrogen ion (a "molecule" μpe consisting of a proton, an electron and a muon) with the muon and electron being in circular states. We showed that this is indeed possible. We demonstrated that in this case, the muonic motion can represent a rapid subsystem while the electronic motion – a slow subsystem. We used a classical analytical description to find the energy terms for the quasimolecule where the muon rotates around the axis connecting the immobile proton and the immobile electron, i.e., dependence of the energy of the muon on the distance between the proton and electron. We found that there is a double-degenerate energy term. We demonstrated that it corresponds to a stable motion. Then we unfroze the slow subsystem and analysed a slow revolution of the axis connecting the proton and electron. We derived the condition required for the validity of the separation into the rapid and slow subsystems. Finally we showed that the spectral lines, emitted by the muon in the quasimolecule μpe, experience a red shift compared to the corresponding spectral lines that would have been emitted by the muon in a muonic hydrogen atom (in the μp-subsystem). The relative values of this red shift, which is a "molecular" effect, are significantly greater than the resolution of available spectrometers and thus can be observed. Observing this red shift should be one of the ways to detect the formation of such muonic negative hydrogen ions.

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1. INTRODUCTION

Studies of the *epe*-system (electron-proton-electron), a.k.a. the negative ion of hydrogen H^-, constitute an important line of research in atomic physics and astrophysics. It has only one bound state – the ground state having a relatively small bound energy of approximately 0.75 eV. This *epe*-system exhibits rich physics. Correlations between the two electrons are strong already in the ground state. With long-range Coulomb interactions between all three pairs of particles, the dynamics is particularly subtle in a range of energies 2 – 3 eV on either side of the threshold for break-up into proton + electron + electron at infinity [1]. There are strong correlations in energy, angle, and spin degrees of freedom, so that perturbation theory and other similar methods fail [1]. Experimental studies of H^- provided a testing ground for the theory of correlated multielectron systems. Compared to the helium atom, the structure of H^- is even more strongly influenced by interelectron repulsion because the nuclear attraction is smaller for this system [2]. In addition to the above fundamental importance, the rich physics of H^- is also important in studies of the ionosphere's D-layer of the Earth atmosphere, the atmosphere of the Sun and other stars, and in development of particle accelerators [1].

Another line of research is studies of muonic atoms and molecules, where one of the electrons is substituted by the heavier lepton μ^-. This line of research has several applications. The first one is muon-catalyzed fusion (see, e.g., [3-5] and references therein). When a muon replaces the electron either in the *dde*-molecule (D_2^+), which becomes the $dd\mu$-molecule, or in the *dte*-molecule, which becomes the $dt\mu$-molecule, the equilibrium internuclear distance becomes about 200 times smaller. At such small internuclear distances, the fusion can occur with a significant probability, which has been observed in $dd\mu$ or even with a higher rate in $dt\mu$ [3-5]. The second application is a laser-control of nuclear processes. This has been discussed in the context of the interaction of muonic molecules with superintense laser fields [6]. Another application is a search for strongly interacting massive particles (SIMPs) proposed as dark matter candidates and as candidates for the lightest supersymmetric particle (see, e.g., [7] and references therein). SIMPs could bind to the nuclei of atoms, and would manifest themselves as anomalously heavy isotopes of known elements. By greatly increasing the nuclear mass, the presence of a SIMP in the nucleus effectively eliminates the well-known reduced mass correction in a hydrogenic atom. Muonic atoms are better candidates (than electronic atoms) for observing this effect because the muon's much larger mass (compared to the electron) amplifies the reduced mass correction [7]. This may be detectable in astrophysical objects [7].

In the present paper we combine the above two lines of research: studies of negative hydrogen ion and studies of muonic atoms/molecules. Namely, we consider a muonic negative hydrogen ion, i.e. μpe-system. Specifically, we study the possibility of circular states in such a system. We show that the muonic motion can represent a rapid subsystem, while the electronic motion – a slow subsystem.

First, we find analytically classical energy terms for the rapid subsystem at the frozen slow subsystem, i.e., for the quasimolecule where the muon rotates around the axis connecting the immobile proton and the immobile electron. The meaning of classical energy terms is explained below. We demonstrate that the muonic motion is stable.

Then we unfreeze the slow subsystem and analyse a slow revolution of the axis connecting the proton and electron. We derive the condition required for the validity of the separation into the rapid and slow subsystems.

Finally we show that the spectral lines, emitted by the muon in the quasimolecule μpe, experience a red shift compared to the corresponding spectral lines that would have been emitted by the muon in a muonic hydrogen atom (in the μp-subsystem). Observing this red shift should be one of the ways to detect the formation of such muonic negative hydrogen ions.

2. ANALYTICAL SOLUTION FOR CLASSICAL ENERGY TERMS OF THE RAPID SUBSYSTEM

We consider a hydrogen atom with a muon rotating in a circle perpendicular to and centered at the axis connecting the proton and the electron – see Fig. 1. As we show below, in this configuration the muon may be considered the rapid subsystem while the proton and electron will be the slow subsystem, which essentially reduces the problem to the two stationary Coulomb center problem, where the effective stationary "nuclei" will be the proton and electron. The straight line connecting the proton and electron will be called here "internuclear" axis. We use the atomic units in this study.

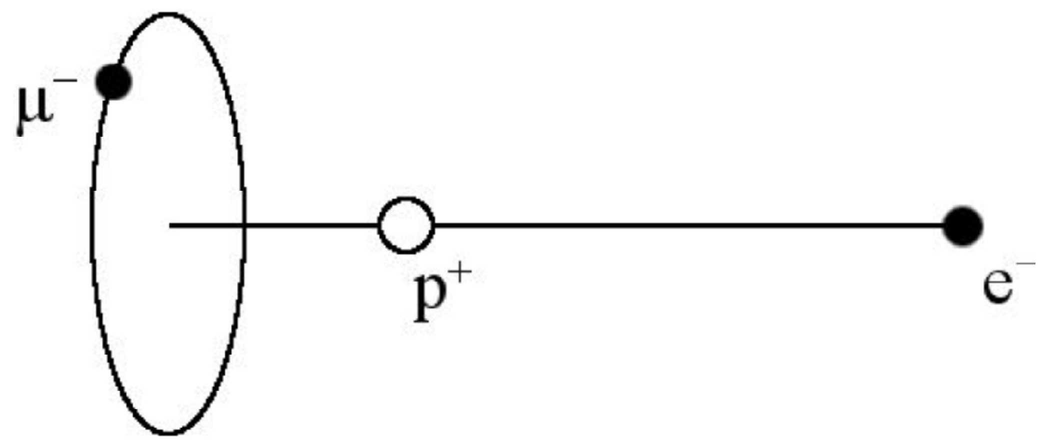

Figure 1: A muon rotating in a circle perpendicular to and centered at the axis connecting the proton and the electron

A detailed classical analytical solution of the two stationary Coulomb center problem, where an electron revolves around nuclei of charges Z and Z', has been presented in papers [8, 9]. We base our results in part on the results obtained in [8, 9].

The Hamiltonian of the rotating muon is

$$H = (p_z^2 + p_\rho^2 + p_\varphi^2/\rho^2)/(2m) - Z/(z^2 + \rho^2)^{1/2} - Z'/[(R - z)^2 + \rho^2]^{1/2}, \quad (1)$$

where m is the mass of the muon (in atomic units $m = 206.7682746$), Z and Z' are the charges of the effective nuclei (in our case, $Z = 1$ and $Z' = -1$), R is the distance between the effective nuclei, (ρ, φ, z) are the cylindrical coordinates, in which Z is at the origin and Z' is at $z = R$, and (p_ρ, p_φ, p_z) are the corresponding momenta of the muon.

Since φ is a cyclic coordinate, the corresponding momentum is conserved:

$$|p_\varphi| = \text{const} = L. \tag{2}$$

With this substituted into Eq. (1), we obtain the Hamiltonian for the z- and ρ-motions

$$H_{z\rho} = (p_z^{\,2} + p_\rho^{\,2})/2 + U_{\text{eff}}(z, \rho), \tag{3}$$

where an effective potential energy is:

$$U_{\text{eff}}(z, \rho) = L^2/(2m\rho^2) - Z/(z^2 + \rho^2)^{1/2} - Z'/[(R - z)^2 + \rho^2]^{1/2}. \tag{4}$$

Since in a circular state $p_z = p_\rho = 0$, the total energy $E(z, \rho) = U_{\text{eff}}(z, \rho)$.

With $Z = 1$, $Z' = -1$ and the scaled quantities

$$w = z/R,\ v = \rho/R,\ \varepsilon = -ER,\ \ell = L/(mR)^{1/2},\ r = mR/L^2, \tag{5}$$

we obtain the scaled energy ε of the muon:

$$\varepsilon = 1/(w^2 + v^2)^{1/2} - 1/[(1 - w)^2 + v^2]^{1/2} - \ell^2/(2v^2) \tag{6}$$

The equilibrium condition with respect to the scaled coordinate w is $\partial\varepsilon / \partial w = 0$; the result can be brought to the form:

$$[(1 - w)^2 + v^2]^{3/2}/(w^2 + v^2)^{3/2} = (w - 1)/w. \tag{7}$$

Since the left side of Eq. (7) is positive, the right side must be also positive: $(w - 1)/w > 0$. Consequently, the allowed ranges of w here are $-\infty < w < 0$ and $1 < w < +\infty$. This means that equilibrium positions of the center of the muon orbit could exist (judging only by the equilibrium with respect to w) either beyond the proton or beyond the electron, but there are no equilibrium positions between the proton and electron.

Solving Eq. (7) for v^2 and denoting $v^2 = p$, we obtain:

$$p(w) = w^{2/3}(w - 1)^{2/3}[w^{2/3} + (w - 1)^{2/3}]. \tag{8}$$

The equilibrium condition with respect to the scaled coordinate v is $\partial\varepsilon / \partial v = 0$, which yields:

$$\ell^2 = p^2\{1/(w^2 + p)^{3/2} - 1/[(1 - w)^2 + p]^{3/2}\}. \tag{9}$$

Since the left side of Eq. (9) is positive, the right side must be also positive. This entails the relation $w^2 + p < (1 - w)^2 + p$, which simplifies to $2w - 1 < 0$, which requires $w < 1/2$.

Thus, the equilibrium with respect to both w and v is possible only in the range $-\infty < w < 0$, while in the second range, $1 < w < +\infty$ (derived from the equilibrium with respect to w only) there is no equilibrium with respect to v.

From the last two relations in Eq. (5), we find $r = 1 / \ell^2$; thus

$$r = p^{-2}\{1/(w^2 + p)^{3/2} - 1/[(1 - w)^2 + p]^{3/2}\}^{-1}, \tag{10}$$

where p is given by Eq. (8). Therefore, the quantity r in Eq. (10) is the scaled "internuclear" distance dependent on the scaled internuclear coordinate w.

Now we substitute the value of ℓ from Eq. (9), as well as the value of p from Eq. (8) into Eq. (6), obtaining $\varepsilon(w)$ – the scaled energy of the muon dependent on the scaled internuclear coordinate w. Since $E = -\varepsilon/R$ and $R = rL^2/m$, then $E = -(m/L^2)\varepsilon_1$ where $\varepsilon_1 = \varepsilon/r$. The parametric dependence $\varepsilon_1(r)$ will yield the energy terms.

The form of the parametric dependence $\varepsilon_1(r)$ can be significantly simplified by introducing a new parameter $\gamma = (1 - 1/w)^{1/3}$. The region $-\infty < w < 0$ corresponds to $1 < \gamma < \infty$. The parametric dependence will then have the following form:

$$\varepsilon_1(\gamma) = (1-\gamma)^4(1+\gamma^2)^2/\,[2(1-\gamma+\gamma^2)^2(1+\gamma^2+\gamma^4)], \tag{11}$$

$$r(\gamma) = (1+\gamma^2+\gamma^4)^{3/2}/\,[\gamma(1+\gamma^2)^2], \tag{12}$$

Classical energy terms given by the parametric dependence of the scaled energy $\varepsilon_1 = (L^2/m)E$ on the scaled internuclear distance $r = (m/L^2)R$ are presented in Fig. 2.

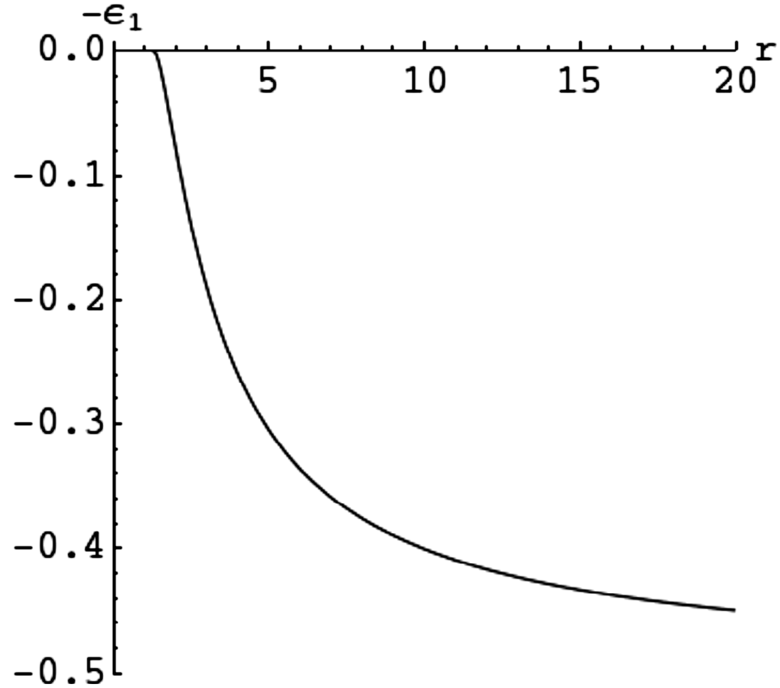

Figure 2: Classical energy terms: the scaled energy $-\varepsilon_1 = (L^2/m)E$ versus the scaled internuclear distance $r = (m/L^2)R$

Figure 2 actually contains two coinciding energy terms: there is a double degeneracy with respect to the sign of the projection of the muon angular momentum on the internuclear axis. We remind the readers that L is the absolute value of this projection – in accordance to its definition in Eq. (2).

The minimum value of R, corresponding to the point where the term starts, can be found from Eq. (12). The term starts at $w = -\infty$, which corresponds to $\gamma = 1$; taking the value of (12) at this point, we find

$$R_{\min} = (3^{3/2}/4)L^2/m. \tag{13}$$

With the value of $m = 206.7682746$, Eq. (13) yields $R = 0.00628258\ L^2$.

The following note might be useful. The plot in Fig. 2 represents two degenerate classical energy terms of “the same symmetry”. (In physics of diatomic molecules, the terminology “energy terms of the same symmetry” means the energy terms of the same projection of the angular momentum on the internuclear axis.) For a given R and L, the classical energy E takes only one *discrete* value. However, as L varies over a *continuous* set of values, so does the classical energy E (as it should be in classical physics).

The revolution frequency of the muon Ω is

$$\Omega = L/(m\rho^2) = L/(mR^2v^2) = L/(mR^2p) \tag{14}$$

in accordance to the previously introduced notations $p = v^2 = (\rho/R)^2$. Since $R = L^2r/m$ (see Eq. (5)), then Eq. (14) becomes $\Omega = (m/L^3)f$, where $f = 1/(pr^2)$. Using Eq. (12) for $r(\gamma)$ and Eq. (8) for $p(w)$ with the substitution $w = 1/(1-\gamma^3)$, where $\gamma > 1$, we finally obtain:

$$\Omega = (m/L^3)\,f(\gamma), \quad f(\gamma) = (1+\gamma^2)^3(1-\gamma^3)^2/(1+\gamma^2+\gamma^4)^3, \tag{15}$$

where $f(\gamma)$ is the scaled muon revolution frequency. Fig. 3 shows the scaled muon revolution frequency $f = (L^3/m)\omega$ versus the scaled internuclear distance $r = (m/L^2)R$.

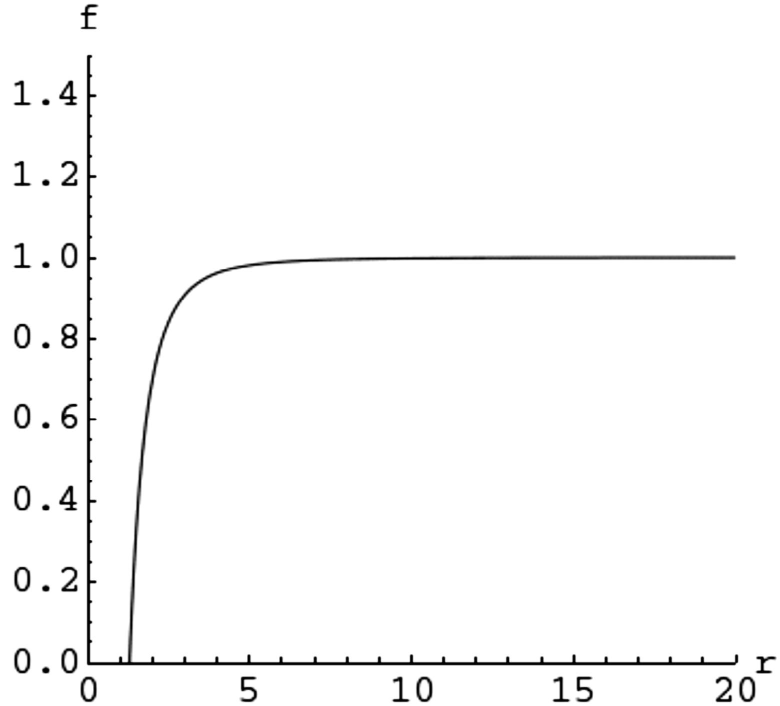

Figure 3: The scaled muon revolution frequency $f = (L^3/m)\Omega$ versus the scaled internuclear distance $r = (m/L^2)R$

It is seen that for almost all values of the scaled internuclear distance $r = (m/L^2)R$, the scaled muon revolution frequency $f = (L^3/m)\Omega$ is very close to its maximum value $f_{max} = 1$, corresponding to large values of R. (The quantity f_{max} can be easily found from Eq. (15) given that large values of R correspond to $\gamma >> 1$ and that this limit yields $f_{max} = 1$.) In other words, for almost all values of R, the muon revolution frequency Ω is very close to its maximum value

$$\Omega_{max} = m/L^3. \qquad (16)$$

In Sect. 3, we will compare the muon revolution frequency with the corresponding frequency of the electronic motion and derive the condition of validity of the separation into rapid and slow subsystems.

For analysing the stability of the muon motion, corresponding to the degenerate classical energy terms, we use the same approach as in paper [9]. Namely, in paper [9], while considering a classical circular motion of a charged particle (which was the electron in [9]) in the field of two stationary Coulomb centers, using the same notations as in the present paper, it was shown that the frequencies of small oscillations of the scaled coordinates w and v of the circular orbit around its equilibrium position are given by

$$\omega_{\pm} = [1/(1 - w) \pm 3w/Q]^{1/2}/(w^2 + p)^{3/4} \qquad (17)$$

where

$$Q = (w^2 + p)^{1/2}\,[(1 - w)^2 + p]^{1/2} \qquad (18)$$

These oscillations are in the directions (w', v') obtained by rotating the (w, v) coordinates by the angle α:

$$\delta w' = \delta w \cos\alpha + \delta v \sin\alpha \quad \delta v' = -\,\delta w \sin\alpha + \delta v \cos\alpha \qquad (19)$$

where the "δ" symbol stands for the small deviation from equilibrium. The angle α is determined by the following relation:

$$\alpha = ½ \tan^{-1}\{(1 - 2w)p^{1/2}/[w(1 - w) + p]\} \qquad (20)$$

The quantity Q in (18) is always positive since it contains the squares of the coordinates. From Eq. (17) it is seen that the condition for both frequencies to be real is

$$1 / (1 - w) \geq 3w/Q \tag{21}$$

For the frequency ω_- to be real, Eq. (17) requires $Q \geq 3w(1 - w)$. For any $w < 0$ (which is the allowed range of w), this inequality is satisfied: the left side is always positive while the right side is always negative.

For the frequency ω_+ to be real, the following function $F(w)$ must be positive (in accordance to Eqs. (17), (18)):

$$F(w) = (w^2 + p)\,[(1 - w)^2 + p] - 9w^2(1 - w)^2 \tag{22}$$

After replacing w by $\gamma = (1 - 1/w)^{1/3}$, the expression (22) becomes

$$F(\gamma) = \gamma^2(\gamma^2 - 1)^2(1 + 4\gamma^2 + \gamma^4) / (\gamma^3 - 1)^4 \tag{23}$$

Since the allowed range of $w < 0$ corresponds to $\gamma > 1$, it is seen that $F(\gamma)$ is always positive.

Thus, the corresponding classical energy terms correspond to the stable motion.

3. ELECTRONIC MOTION AND THE VALIDITY OF THE SCENARIO

Now we unfreeze the slow subsystem and analyse a slow revolution of the axis connecting the proton and electron, the electron executing a circular orbit. In accordance to the concept of separating rapid and slow subsystems, the rapid subsystem (the revolving muon) follows the adiabatic evolution of the slow subsystem. This means that the slow subsystem can be treated as a modified "rigid rotator" consisting of the electron, the proton, and the ring, over which the muon charge is uniformly distributed, all distances within the system being fixed (see Fig. 1).

The potential energy of the electron in atomic units (with the angular-momentum term) is

$$E_e = M^2/(2R^2) - 1/R + 1/[\rho^2 + (R - z)^2]^{1/2} \tag{24}$$

where M is the electronic angular momentum. Its derivative by R must vanish at equilibrium, which yields

$$dE_e/dR = -M^2/R^3 + 1/R^2 - (R - z)/[\rho^2 + (R - z)^2]^{3/2} = 0 \tag{25}$$

which gives us the value of the scaled angular momentum

$$\ell_e = M/R^{1/2} \tag{26}$$

corresponding to the equilibrium:

$$\ell_e^2 = 1 - (1 - w)/[(1 - w)^2 + p]^{3/2} \tag{27}$$

where the scaled quantities w, p of the muon coordinates are defined in Eq. (5). Using the muon equilibrium condition from Eq. (7) with v^2 denoted as p, we can represent Eq. (27) in the form:

$$\ell_e^2 = 1 + w/(w^2 + p)^{3/2}. \tag{28}$$

After replacing w by $\gamma = (1 - 1/w)^{1/3}$, we obtain

$$\ell_e(\gamma) = [1 - (1 - \gamma)^2(1 + \gamma + \gamma^2)^{1/2}/(1 - \gamma + \gamma^2)^{3/2}]^{1/2} \tag{29}$$

The electron revolution frequency is $\omega = M/R^2 = \ell_e(\gamma)/R^{3/2}$ given that $M = \ell_e(\gamma)R^{1/2}$ in accordance to Eq. (26). Since $R = L^2r(\gamma)/m$ (see Eq. (5)) with $r(\gamma)$ given by Eq. (12), then from $\omega = \ell_e(\gamma) / R^{3/2}$ we obtain

$$\omega = m^{3/2}\,\ell_e(^3)/\{L^3[r(\gamma)]^{3/2}\}. \tag{30}$$

From Eqs. (15) and (30) we find the following ratio of the muon and electron revolution frequencies:

$$\Omega / \omega = (1/m^{1/2})\; f(\gamma)[r(\gamma)]^{3/2} / \ell_e(\gamma), \tag{31}$$

where $f(\gamma)$ is given in Eq. (15).

In addition to the above relation $R = L^2r(\gamma)/m$, the same quantity R can be expressed from Eq. (26) as $R = M^2/[\ell_e(\gamma)]^2$. Equating the right sides of these two expressions, we obtain the equality $L^2r(\gamma)/m = M^2/[\ell_e(\gamma)]^2$, from which it follows:

$$L/M = m^{1/2}/\{\ell_e(\gamma)[r(\gamma)]^{1/2}\}. \quad (32)$$

The combination of Eqs. (31) and (32) represents an analytical dependence of the ratio of the muon and electron revolution frequencies Ω/ω versus the ratio of the muon and electron angular momenta L/M via the parameter γ as the latter varies from 1 to ∞. This dependence is presented in Fig. 4.

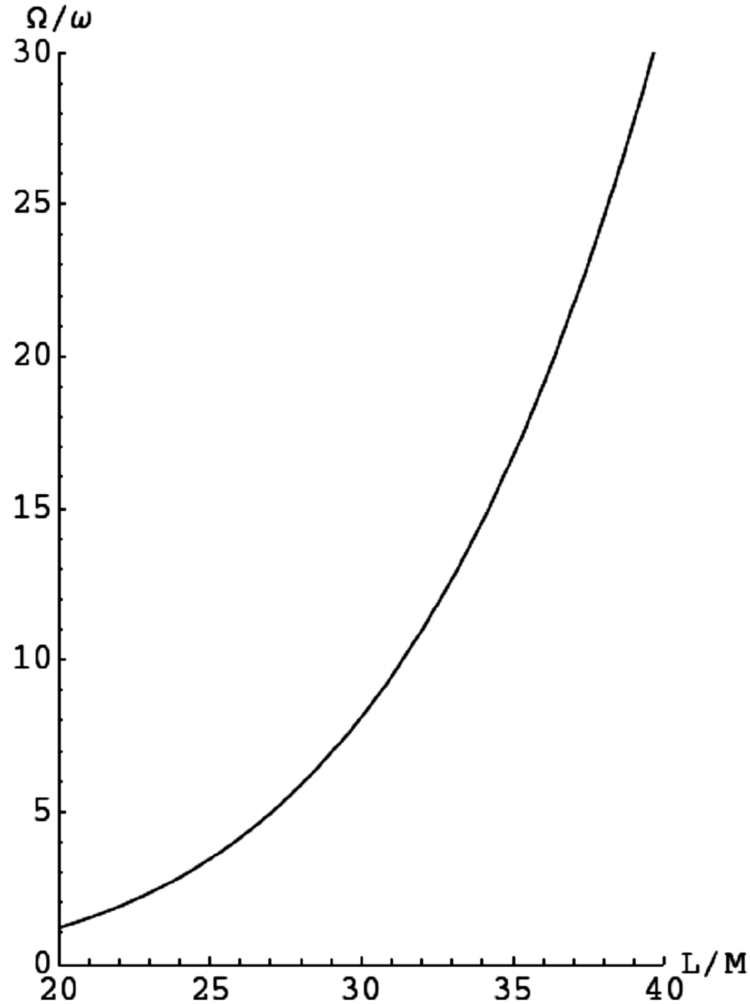

Figure 4: The ratio of the muon and electron revolution frequencies Ω/ω versus the ratio of the muon and electron angular momenta *L/M*

For the separation into the rapid and slow subsystems to be valid, the ratio of frequencies Ω/ω should be significantly greater than unity. From Fig. 4 it is seen that this requires the ratio of angular momenta L/M to be noticeably greater than 20.

There is another validity condition to be checked for this scenario. Namely, the revolution frequency Ω of the muon must be also much greater than the inverse lifetime of the muon $1/T_{life}$, where $T_{life} = 2.2$ μs $= 0.91 \times 10^{11}$ a.u.: $\Omega T_{life} >> 1$. Since for almost all values of R, the muon revolution frequency Ω is very close to its maximum value $\Omega_{max} = m/L^3$, as shown in Sect. 2, then the second validity condition can be estimated as $(m/L^3)T_{life} >> 1$, from which it follows

$$L << L_{max} = (m\ T_{life})^{1/3} = 26600 \quad (33)$$

(we remind that m = 206.7682746 in atomic units). So, the second validity condition is fulfilled for any practically feasible value of the muon angular momentum L.

Thus, for the ratio of angular momenta L/M noticeably greater than 20, we deal here with a muonic quasimolecule where the muon rapidly rotates about the axis connecting the proton and electron following a relatively slow rotation of this axis.

We also performed simulations of the muonic and electronic motion by solving numerically the corresponding Newton's equation. Three samples of the results are presented in Fig. 5 for three different sets of initial conditions. It is seen that the muon trajectory is a spiral resulting from a rapid circular motion of the muon around the axis directed to the electron, while this axis slowly rotates following the motion of the electron along the circular arc.

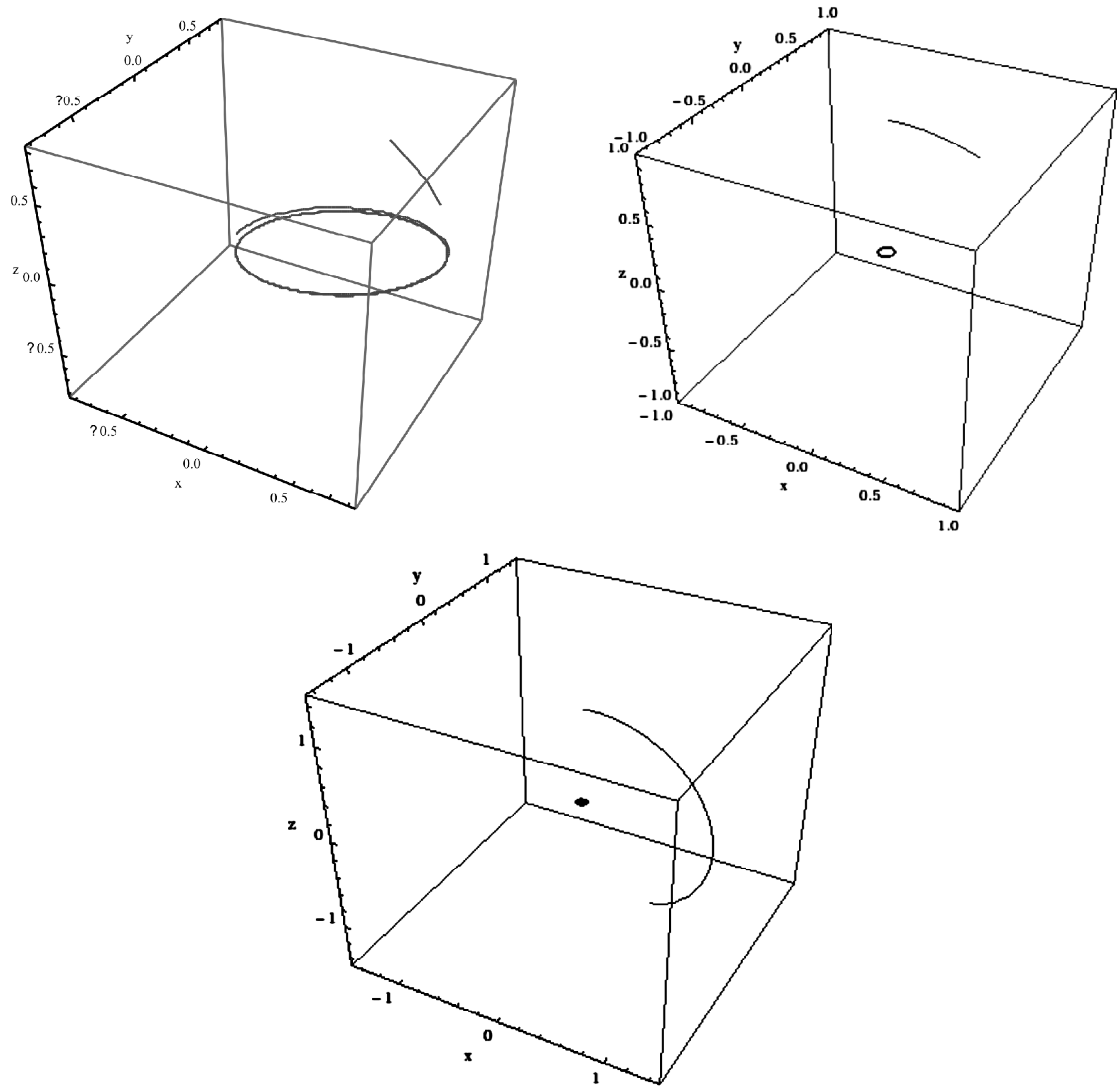

Figure 5: Simulations of the muonic (spiral) and electronic (arc) motion for three different sets of initial conditions

4. RED SHIFT OF SPECTRAL LINES COMPARED TO MUONIC HYDROGEN ATOMS

The muon, rotating in a circular orbit at the frequency $\Omega(R)$, should emit a spectral line at this frequency. The maximum value $\Omega_{max} = m/L^3$ corresponds to the frequency of spectral lines emitted by the muonic hydrogen atom (by the μp-subsystem). For the equilibrium value of the proton-electron separation – just as for almost all values of R – the frequency Ω is slightly smaller than Ω_{max}. Therefore, the spectral lines, emitted by the muon in the quasimolecule μpe, experience a red shift compared to the corresponding spectral lines that would have been emitted by the muon in a muonic hydrogen atom. The relative red shift δ is defined as follows

$$\delta = (\lambda - \lambda_0) / \lambda_0 = (\Omega_{max} - \Omega) / \Omega, \tag{34}$$

where λ and λ_0 are the wavelengths of the spectral lines for the quasimolecule μpe and the muonic hydrogen atom, respectively. Using Eq. (15), the relative red shift can be represented in the form

$$\delta(\gamma) = 1 / f(\gamma) - 1, \tag{35}$$

where $f(\gamma)$ is given in Eq. (15).

The combination of Eqs. (35) and (32) represents an analytical dependence of the relative red shift δ on the ratio of the muon and electron angular momenta L/M via the parameter γ as the latter varies from 1 to ∞. Figure 6 presents the dependence of δ on $L/(m^{1/2}M)$. In this form the dependence is "universal", i.e., valid for different values of the mass m: for example, it is valid also for the quasimolecule πpe where there is a pion instead of the muon. Figure 7 presents the dependence of δ on L/M specifically for the quasimolecule μpe.

It is seen that the relative red shift of the spectral lines is well within the spectral resolution $\Delta\lambda_{res} / \lambda$ of available spectrometers: $\Delta\lambda_{res} / \lambda \sim (10^{-4} - 10^{-5})$ as long as the ratio of the muon and electron angular momenta $L/M < 80$. Thus, this red shift can be observed and this would be one of the ways to detect the formation of such muonic negative hydrogen ions.

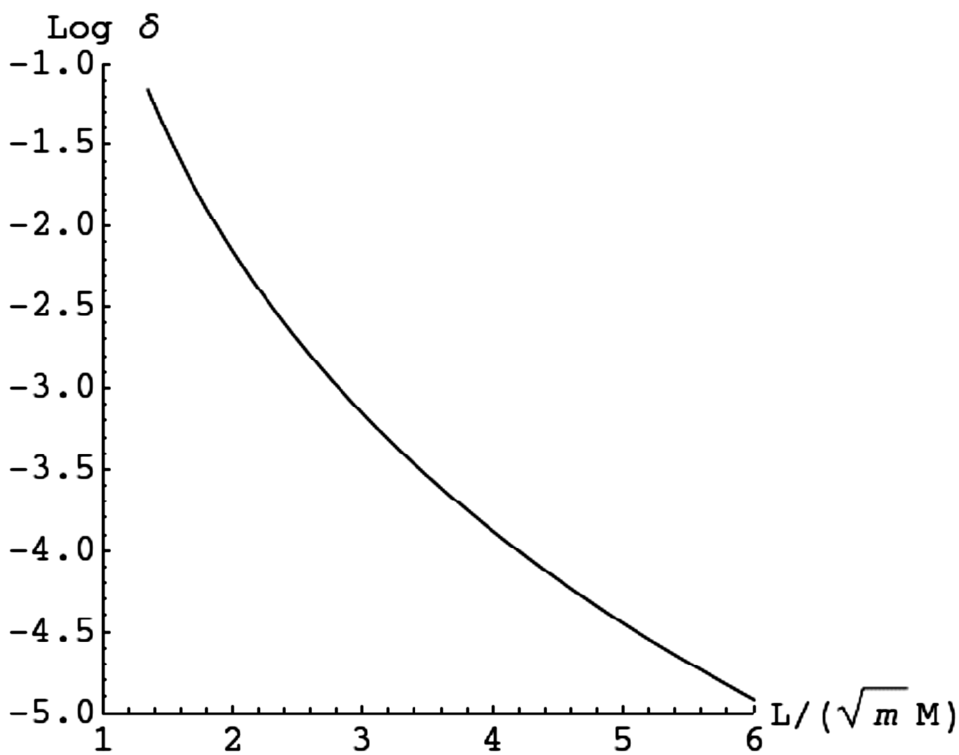

Figure 6: Universal dependence of the relative red shift δ of the spectral lines of the quasimolecule μpe (or πpe) on $L/(m^{1/2}M)$, which is the ratio of the muon and electron angular momenta L/M divided by the square root of the mass m of the muon or pion

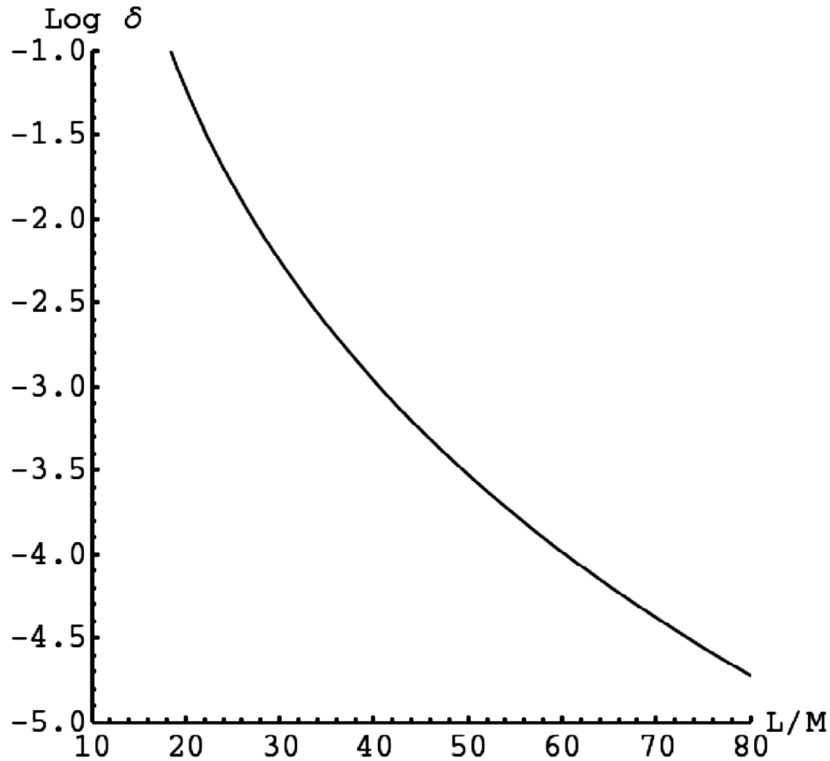

Figure 7: Dependence of the relative red shift δ of the spectral lines of the quasimolecule *μpe* on the ratio of the muon and electron angular momenta *L*/*M*

Figure 8 presents the dependence of the relative red shift δ on the ratio of the muon and electron revolution frequencies Ω/ω. It is seen that the relative red shift decreases as the ratio of the muon and electron revolution frequencies increases, but it remains well within the spectral resolution $\Delta\lambda_{res} / \lambda$ of available spectrometers.

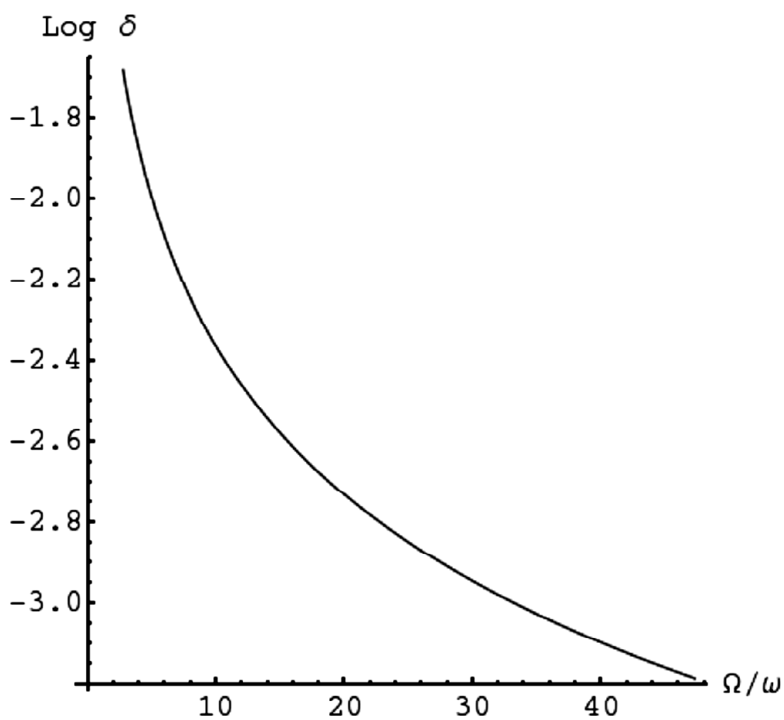

Figure 8: Dependence of the relative red shift δ on the ratio of the muon and electron revolution frequencies Ω/ω

5. CONCLUSIONS

We studied the existence of a muonic negative hydrogen ion (a "molecule" μpe consisting of a proton, an electron and a muon) with the muon and electron being in circular states. We showed that this is indeed possible. We demonstrated that in this case, the muonic motion can represent a rapid subsystem while the electronic motion – a slow subsystem. In other words, the muon rapidly revolves in a circular orbit about the axis connecting the proton and electron while this axis slowly rotates following a relatively slow revolution of the electron around the proton.

We used a classical analytical description to find the energy terms of such a system, i.e., dependence of the energy of the muon on the distance between the proton and electron. We found that there is a double-degenerate energy term. We demonstrated that it corresponds to a stable motion. Then we unfroze the slow subsystem and analysed a slow revolution of the axis connecting the proton and electron. We derived the condition required for the validity of the separation into the rapid and slow subsystems.

Finally we showed that the spectral lines, emitted by the muon in the quasimolecule μpe, experience a red shift compared to the corresponding spectral lines that would have been emitted by the muon in a muonic hydrogen atom (in the μp-subsystem). The relative values of this red shift, which is a "molecular" effect, are significantly greater than the resolution of available spectrometers and thus can be observed. Observing this red shift should be one of the ways to detect the formation of such muonic negative hydrogen ions.

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