I R A M P
8(2), December 2017, pp. 61-72

ISSN: 2229-3159

Effect of Helical Trajectories of Electrons in Strongly Magnetized Plasmas on the Width of Hydrogen/Deuterium Spectral Lines: Analytical Results and Applications to White Dwarfs

EUGENE OKS

Physics Department, 206 Allison Lab., Auburn University, Auburn, AL 36849, USA

ABSTRACT: We consider the effect of *helical trajectories* of perturbing electrons on the width of Hydrogen or Deuterium Spectral Lines (HDSL). We concentrate on the case of a strong magnetic field B, such that the so-called non-adiabatic Stark width practically vanishes and only the so-called adiabatic Stark width remains. Such strong magnetic fields encountered, e.g., in white dwarfs. We calculate analytically the adiabatic Stark width for this case and compare it with the adiabatic Stark width for the rectilinear trajectories of perturbing electrons. We show that the adiabatic Stark width calculated with the allowance for helical trajectories of perturbing electrons does not depend on the magnetic field if the magnetic field is sufficiently strong. We demonstrate that, depending on the particular HDSL and on plasma parameters, the adiabatic Stark width, calculated with the allowance for helical trajectories of perturbing electrons, can be either *by orders of magnitude smaller*, or of the same order, or several times higher than the adiabatic Stark width, calculated for rectilinear trajectories of perturbing electrons. We show that for the range of plasma parameters typical for DA white dwarfs, the neglect for the actual, helical trajectories of perturbing electrons can lead to the overestimation of the Stark width by up to one order of magnitude for the alpha- and beta-lines of the Lyman and Balmer series, or to the underestimation of the Stark width by several times for the delta- and higher-lines of the Balmer series. Therefore, our results should motivate astrophysicists for a *very significant revision* of all existing calculations of the broadening of hydrogen lines in DA white dwarfs. The experimental/observational studies, for which the effect of helical trajectories of perturbing electrons on the Stark width might be significant, are not limited by white dwarfs, but can be performed in a variety of laboratory and astrophysical plasmas emitting the hydrogen or deuterium Ly-alpha line.

Key words: strongly magnetized plasmas; white dwarfs; Stark broadening; helical trajectories; adiabatic width

1. INTRODUCTION

In paper [1] there was developed a general framework for calculating shapes of Hydrogen or Deuterium Spectral Lines (HDSL) in strongly-magnetized plasmas with the allowance for helical trajectories of perturbing electrons. It was shown that in this situation the primary effects manifests by the non-vanishing first order term $\Phi^{(1)}(B)$ of the Dyson expansion of the electron broadening operator – in distinction to the case of rectilinear trajectories, where the first non-vanishing term appeared only in the second order. By using the Ly_α line as an example, it was demonstrated that the shape of *each* of the two σ-components can become a doublet: in addition to the shifted component, there can appear also an unshifted component. Moreover, the shape of *each* of the two σ-components can also become a triplet: in addition to the shifted and unshifted component, there can appear also a component shifted to the opposite wing of the line. Both the positions and the intensities of the shifted components depend strongly on the magnitude of $\Phi^{(1)}(B)$. The primary effect in the entire spectral line was a significant increase of the ratio of the intensity of the central peak to the intensity of any of the two lateral peaks.

In the present paper we consider the *direct effect* of helical trajectories of perturbing electrons *on the width* of HDSL. The focus is at the case of a strong magnetic field B, such that the so-called non-adiabatic Stark width practically vanishes and only the so-called adiabatic Stark width remains. Such strong magnetic fields encountered, e.g., in white dwarfs. We calculate analytically the adiabatic Stark width for this case and compare it with the adiabatic Stark width for the rectilinear trajectories of perturbing electrons, the latter being relevant to the case of vanishingly small magnetic fields. We show that the adiabatic Stark width calculated with the allowance for helical trajectories of perturbing electrons does not depend on the magnetic field if the magnetic field is sufficiently strong.

We demonstrate that, depending on the particular HDSL and on plasma parameters, the adiabatic Stark width, calculated with the allowance for helical trajectories of perturbing electrons, can be either *by orders of magnitude smaller*, or of the same order, or several times higher than the adiabatic Stark width, calculated for rectilinear trajectories of perturbing electrons. We show that for the range of plasma parameters typical for DA white dwarfs (i.e, for white dwarfs emitting hydrogen lines), the neglect for the actual, helical trajectories of perturbing electrons can lead to the overestimation of the Stark width by up to one order of magnitude for the alpha- and beta-lines of the Lyman and Balmer series, or to the underestimation of the Stark width by several times for the delta- and higher-lines of the Balmer series. Therefore, our results should motivate astrophysicists for a very significant revision of all existing calculations of the broadening of hydrogen lines in DA white dwarfs.

We also explain that experimental/observational studies, for which the effect of helical trajectories of perturbing electrons on the Stark width might be significant, are not limited by white dwarfs, but can be performed in a variety of laboratory and astrophysical plasmas emitting the hydrogen or deuterium Ly-alpha line.

2. ANALYTICAL RESULTS

For hydrogen/deuterium atoms in a strongly magnetized plasma, the radius-vector $\mathbf{R}(t)$ of a perturbing electron and the electric field $\mathbf{E}(t)$ it creates at the location of the radiating atom, can be represented in the form

$$\mathbf{R}(t) = v_z t\mathbf{B}/B + \boldsymbol{\rho}[1 + (r_{Bp}/\rho)\cos(\omega_B t + \varphi)] + \boldsymbol{\rho}\mathrm{x}\mathbf{B}\,[r_{Bp}/(\rho B)]\sin(\omega_B t + \varphi), \quad (1)$$

$$\mathbf{E}(t) = e\mathbf{R}(t)/[R(t)]^3,$$

where the z-axis is chosen along the magnetic field $\mathbf{B}$; $\boldsymbol{\rho}\mathrm{x}\mathbf{B}$ stands for the cross-product (also known as the vector product) of the impact parameter vector $\boldsymbol{\rho}$ and the magnetic field $\mathbf{B}$; e is the electron charge. Here

$$r_{Bp} = v_p/\omega_B, \quad \omega_B = eB/(m_e c), \quad (2)$$

where v_p is the electron velocity in the plane perpendicular to $\mathbf{B}$; ω_B is the Larmor frequency (also known as the cyclotron frequency).

Without the allowance for helical trajectories of perturbing electrons, the effect of the magnetic field on the width of HDSL becomes noticeable where the magnetic field exceeds certain critical value B_{cr}, Let us briefly remind the physical reason for this. In the so-called Conventional Theory (CT) of the Stark broadening of HDSL (which is frequently referred to as Griem's theory – as presented in Kepple-Griem paper [2] and in Griem's book [3]) the electron impact broadening operator Φ_{ab} (where a and b label, respectively, the upper and lower states involved in the radiative transition) is a linear function of the following integral over impact parameters:

$$\int_0^\infty d\rho/\rho. \quad (3)$$

This integral diverges both at large and small ρ, so that in the CT the integration got truncated by some ρ_{min} at the lower limit and by some ρ_{max} at the upper limit. The lower limit ρ_{min} is chosen such as to preserve the unitarity of S-matrices involved in the calculation and is then called Weisskopf radius ρ_{We} (discussed in more detail below). At the absence of the magnetic field, the upper limit ρ_{max} is chosen from the requirement that the characteristic frequency v/ρ of the variation of the electric field of the perturbing ion should exceed the plasma electron frequency ω_{pe} according to the general plasma property to screen electric fields at frequencies lower than ω_{pe}. So, it is chosen ρ_{max}

$= v/\omega_{pe}$, which after substituting the perturbing electron velocity v by the mean thermal velocity v_T becomes the Debye radius ρ_D.

The uniform magnetic field **B** reduces the spherical symmetry of the problem to the axial symmetry. The fundamental consequence is that in this situation the electron broadening operator Φ_{ab} (and the corresponding Stark width) should be subdivided into two distinct parts: the *adiabatic* part $\Phi_{ad,ab}$ and the *nonadiabatic* part $\Phi_{na,ab}$. The adiabatic part is controlled by the component of the electric field (of the perturbing electron) parallel to **B**, while the nonadiabatic part is controlled by the component of the electric field (of the perturbing electron) perpendicular to **B**.

Physically the nonadiabatic contribution to the broadening is due to the virtual transitions between the adjacent Zeeman sublevel separated by $\Delta\omega_B = \omega_B/2$. For the nonadiabatic contribution to be effective, the characteristic frequency v/ρ of the variation of the electric field of the perturbing ion should exceed $\Delta\omega_B$. This leads to the modified upper cutoff for the integral (3):

$$\rho_{max} = \min(v/\omega_{pe}, v/\Delta\omega_B) = \min(v/\omega_{pe}, 2v/\omega_B), \tag{4}$$

which becomes smaller than v/ω_{pe} when

$$\Delta\omega_B = \omega_B/2 > \omega_{pe}. \tag{5}$$

or

$$B > B_{threshold} = 4c(m_e N_e)^{1/2}, \quad B_{threshold}(\text{Tesla}) = 3.62\text{x}10^{-7}[N_e(\text{cm}^{-3})]^{1/2}, \tag{6}$$

where N_e is the electron density. Obviously, the greater the ratio B/B_{cr}, the smaller the nonadiabatic contribution to the width becomes. Such inhibition of the nonadiabatic contribution to the width was studied in detail in paper [4] – see also Sect. 4 of review [5].

In distinction, the adiabatic contribution to the width is not affected by the fulfilment of condition (5) or (6). Physically this is because the adiabatic contribution is not related to the quantum effect of the virtual transitions between the adjacent Zeeman sublevels. Rather, the physics behind the adiabatic contribution is the phase modulation of the atomic oscillator by the parallel to **B** component of the electric field of the perturbing electron (the phase modulation being, in essence, a classical effect – see. e.g., review [6].

Thus, it is clear that as $B/B_{threshold}$ becomes much greater than unity, practically the entire width becomes due to the adiabatic contribution. This happens when $\rho_{max} = v/\Delta\omega_B$ diminishes to the value significantly below $\rho_{min} = \rho_{We}$, i.e., when $\Delta\omega_B > v/\rho_{We}$, which after substituting v by $v_T = (2T_e/m_e)^{1/2}$, T_e being the electron temperature, becomes

$$\Delta\omega_B > \Omega_{We} = v_T/\rho_{We}, \tag{7}$$

where Ω_{We} is called the Weiskopf frequency, or

$$B > B_{cr} = 2m_e c v_T/(e\rho_{We}). \tag{8}$$

The focus of the present paper is at very strong magnetic fields satisfying the condition (8), which will be reformulated more explicitly below. In this situation, the effect of helical trajectories of the perturbing electrons can be presented in the purest form – without the interplay with the magnetic-field-caused inhibition of the nonadiabatic contribution studied in paper [4].

For the purpose of the comparison, let us first calculate the adiabatic contribution for rectilinear trajectories of perturbing electrons in the case of a vanishingly small magnetic field **B** – in the spirit of the CT, but slightly more accurately. By using the parabolic coordinates with the z-axis along **B**, the adiabatic part $(\Phi_{ad,rec})_{ab}$ of the electron broadening operator can be represented in the form (the suffix “rec” stands for “rectilinear trajectories”)

$$(\Phi_{ad,rec})_{ab} = -[2\hbar^2 N_e/(3m_e^2)][(Z_a - Z_b)(Z_a - Z_b)/a_0^2]2\pi < v(1/2 + \int_{\rho_{min}(v)}^{\rho_{max}(v)} d\rho/\rho)/v^2 >_{vel}, \tag{9}$$

where $<\ldots>_{vel}$ stands for averaging over the distribution of velocities of perturbing electrons and 1/2 stands for the contribution of the so-called strong collisions (i.e., the collisions with the impact parameters $\rho < \rho_{min}(v)$. In Eq. (9), Z is the operator of the z-projection of the radius-vector of the atomic electron. In the parabolic coordinates, this operator has only diagonal matrix elements in the manifold of the fixed principal quantum number n, so that the operator $(\Phi_{ad,rec})_{ab}$ also has only diagonal matrix elements, which we denote ${}_{\alpha\beta}(\Phi_{ad,rec})_{\beta\alpha}$. Here α and β correspond, respectively, to upper and lower sublevels of the levels a and b involved in the radiative transition.

Then the adiabatic width $\gamma_{\alpha\beta,rec} = -\mathrm{Re}[{}_{\alpha\beta}(\Phi_{ad,rec})_{\beta\alpha}]$ can be expressed as follows

$$\gamma_{\alpha\beta,rec} = [3\hbar^2 N_e X_{\alpha\beta}{}^2/(2m_e{}^2)]2\pi <v\,[(1/2 + \int_{\rho_{min}(v)}^{\rho_{max}(v)} d\rho/\rho)/v^2]>_{vel}, \tag{10}$$

where

$$X_{\alpha\beta} = n_a q_\alpha - n_b q_\beta, \quad q_\alpha = (n_1 - n_2)_\alpha, \ q_\alpha = (n_1 - n_2)_\alpha, \tag{11}$$

where n_1 and n_2 are the parabolic quantum numbers (while q is often called the electric quantum number). We note that $\gamma_{\alpha\beta,rec}$ is the half-width at half maximum of the corresponding component of the line. We also note that $X_{\alpha\beta}$ is the standard label of Stark components of HDSL, but for avoiding any confusion we emphasize that in the present paper we consider the Zeeman triplet of HDSL, consisting of the central (unshifted) π-component and two σ-components symmetrically shifted to the red and blue parts of the line profile.

For the adiabatic width we have $\rho_{max}(v) = v/\omega_{pe}$, as explained above. As for $\rho_{min}(v)$, its role is played by the *adiabatic Weisskopf radius*

$$\rho_{Wad}(v) = k_{ad}/v, \quad k_{ad} = 3|X_{\alpha\beta}|\hbar/m_e. \tag{12}$$

The adiabatic Weisskopf radius arises naturally without any uncertainty as a result of the *exact* calculation of the adiabatic Stark width in paper [7]. This is the primary distinction of the adiabatic Weisskopf radius from the Weisskopf radius ρ_{WG} in Griem's theory. In the latter it was chosen from the requirement to preserve the unitarity of the S-matrices entering the calculation:

$$|1 - S_a(\rho, v)\, S_b{}^*(\rho, v)| \le 2, \tag{13}$$

where the symbol * stands for the complex conjugation.

Therefore, the choice of ρ_{WG} in Griem's theory

$$\rho_{WG}(v) = k_G/v, \qquad k_G = \hbar(n_a{}^2 - n_b{}^2)/m_e \tag{14}$$

has an inherent uncertainty by a factor of 2 (for the detailed discussion of this uncertainty see, e.g., paper [8] or Appendix B from book [9]). Below, while calculating the velocity average in Eq. (9), we will use

$$\rho_{min}(v) = \rho_W(v) = k/v, \tag{15}$$

where k would be either k_{ad} from Eq. (11) or k_G from Eq. (13), so that for the ratio $\rho_{max}(v)/\rho_{min}(v)$ we have

$$\rho_{max}(v)/\rho_{min}(v) = v^2/(k\omega_{pe}). \tag{16}$$

Then the result of the integration over the impact parameters in Eq. (10) can be expressed as follows

$$\ln[(v^2/v_T{}^2)/D], \qquad D = k\omega_{pe}/v_T{}^2. \tag{17}$$

While using $k = k_{ad}$ defined in Eq. (12), the dimensionless constant D becomes

$$D = 3|X_{\alpha\beta}|\hbar\omega_{pe}/(m_e v_T{}^2) = 3|X_{\alpha\beta}|\hbar\omega_{pe}/(2T_e) = 5.57\mathrm{x}10^{-11}|X_{\alpha\beta}|[N_e(\mathrm{cm}^{-3})]^{1/2}/T_e(\mathrm{eV}). \tag{18}$$

At this point we calculate the velocity average in Eq. (10) by employing the three-dimensional isotropic Maxwell distribution $f_3(p)dp = (4/\pi^{1/2})p^2\exp(-p^2)$, where $p = v/v_T$:

$$<\{\ln[(v^2/v_T^2)/D] + 1/2\}/v]>_{vel} = (2/\pi^{1/2})[\ln(1/D) + 1/2 - \Gamma] = (2/\pi^{1/2})[\ln(1/D) - 0.0772], \quad (19)$$

where $\Gamma = 0.5772$ is the Euler constant. So, finally the adiabatic width in the case of rectilinear trajectories becomes:

$$\gamma_{\alpha\beta,rec} = [6\pi^{1/2}\hbar^2 X_{\alpha\beta}{}^2 N_e/(m_e{}^2 v_T)]\,[\ln(1/D) - 0.0772], \quad v_T = (2T_e/m_e)^{1/2}, \quad (20)$$

where D was defined in Eq. (18).

Now we start calculating the adiabatic width $\gamma_{\alpha\beta,hel}$ for the case of the actual trajectories (i.e., helical trajectories) of perturbing electrons, the suffix "hel" standing for "helical trajectories". Again we consider strong magnetic fields satisfying the condition (8). By using the explicit expression (12) for the adiabatic Weisskopf radius and substituting v by v_T, the condition (8) can be reformulated more explicitly as follows

$$B > B_{cr}, \qquad B_{cr} = 2T_e/(3|X_{\alpha\beta}|e\lambda_c) \text{ or } B_{cr}(\text{Tesla}) = 9.2\text{x}10^2 T_e(\text{eV})/|X_{\alpha\beta}| \text{ Tesla}, \quad (21)$$

where $\lambda_c = \hbar/(m_e c) = 2.426\text{x}10^{-10}$ cm is the Compton wavelength of electrons.

Such strong magnetic fields are encountered, for example, in plasmas of DA white dwarfs (i.e., white dwarfs emitting hydrogen lines). According to observations, the magnetic field in plasmas of white dwarfs can range from 10^3 Tesla to 10^5 Tesla (see, e.g., papers [10, 11]), thus easily exceeding the critical value from Eq. (21) – given that $T_e \sim 1$ eV in the white dwarfs plasmas emitting hydrogen lines.

For completeness it should be mentioned that we consider the situation where the temperature T_a of the radiating atoms satisfies the condition

$$T_a << (11.12 \text{ keV/n})(M/M_H)^2, \quad (22)$$

where M is the mass of the radiating atom, M_H is the mass of hydrogen atoms, and n is the principal quantum number of the energy level, from which the spectral line originates. Under this condition, the Lorentz field effects can be disregarded compared to the "pure" magnetic field effects.

It should be also noted that *at any value of the magnetic field* (no matter how large or small), the Stark width of the central (unshifted) component of the Ly-alpha Zeeman triplet has practically only the adiabatic contribution – because the non-adiabatic contribution vanishes within the accuracy of about 1%, as shown in detail in paper [4]. So, the experimental/observational studies, for which the effect of helical trajectories of perturbing electrons on the Stark width might be significant, are not limited by white dwarfs, but can be performed in a variety of laboratory and astrophysical plasmas emitting the hydrogen or deuterium Ly-alpha line (by using the polarization analysis).

Based on Eq. (1), the z-component (i.e., the component parallel to **B**) of the electric field of the perturbing electron at the location of the radiating atom can be represented in the form

$$E_z(t) = e(v_z t)/[\rho^2 + v_z{}^2 t^2 + v_p{}^2/\omega_B{}^2 + 2(\rho v_p/\omega_B)\cos(\omega_B t + \varphi)]^{3/2}, \quad (23)$$

For comparison, in the case of the rectilinear trajectories for vanishingly small **B** it was

$$E_{z,rec}(t) = e(\boldsymbol{\rho}\mathbf{e}_z + \mathbf{v}\mathbf{e}_z t)/(\rho^2 + v^2 t^2)^{3/2}, \quad (24)$$

where $\mathbf{e}_z$ is the unit vector along the z-axis. Here and below, for any two vectors **a** and **b**, the notation **ab** stands for their scalar product (also known as the dot-product). It should be noted that Eq. (23) in the limit of B = 0 does not reduce to Eq. (24). This is because Eq. (23) adequately describes the situation only for the strong magnetic fields defined by the condition

$$(v_p/\omega_B)^2 << [\rho_{Wad}(v_z)]^2. \quad (25)$$

It can be reformulated as

$$[m_e/(3X_{\alpha\beta}e\lambda_c B)]^2 v_p{}^2 v_z{}^2 << 1, \quad (26)$$

which after substituting v_p^2 by its average value over the two dimensional Maxwell distribution $<v_p^2> = v_T^2$ and v_z^2 by its average value over the one dimensional Maxwell distribution $<v_z^2> = v_T^2/2$, can be rewritten as

$$B >> B_{min} = 2^{1/2}m_e v_T^2/(6|X_{\alpha\beta}|e\lambda_c) = 2^{1/2}T_e/(3|X_{\alpha\beta}|e\lambda_c) \text{ or } B_{min}(\text{Tesla}) = 6.5\text{x}10^2 T_e(\text{eV})/|X_{\alpha\beta}|. \quad (27)$$

This condition is similar to the condition (8) or (21), under which practically the entire width of HDSL becomes due to the adiabatic contribution only.

The starting formula for the adiabatic width in the case of helical trajectories of perturbing electrons $\gamma_{\alpha\beta,hel} = -\text{Re}[_{\alpha\beta}(\Phi_{ad\,,hel})_{\beta\alpha}]$ (the suffix "hel" stands for "helical trajectories")

$$\gamma_{\alpha\beta,hel} = N\int_{-\infty}^{\infty} dv_z\, f_1(v_z)\int_0^{\infty} dv_p\, f_1(v_p)\,(v_z^2 + v_p^2)^{1/2}\sigma(v_z, v_p), \quad (28)$$

where the operator $\sigma(v_z, v_p)$, which is physically the cross-section of the so-called optical collisions responsible for the broadening of the spectral line, has the form:

$$\sigma(v_z, v_p) = \int_0^{\infty} d\rho\, 2\pi\rho[1 - S_a(\rho, v_z, v_p)S_b^*(\rho, v_z, v_p)]_\varphi. \quad (29)$$

Here $f_1(v)$ and $f_2(v)$ are the 1D- and 2D-Maxwell distributions, respectively; S_a and S_b are the corresponding scattering matrices; the symbol $[\ldots]_\varphi$ stands for the average over the phase φ entering Eq. (23). In the spirit of the CT, one gets

$$1 - S_a(\rho, v_z, v_p)S_b^*(\rho, v_z, v_p) = [3X_{\alpha\beta}\hbar/(2m_e e)]^2 \int_{-\infty}^{\infty} dt\, E_z(t)\int_{-\infty}^{t} dt_1\, E_z(t_1), \quad (30)$$

where $E_z(t)$ is given by Eq. (23).

It is important to emphasize the following. The double integral in Eq. (30 vanishes for the odd part of $E_z(t)$, while for the even part of $E_z(t)$ it is equal to the one half of the square of $\int_{-\infty}^{\infty} dt\, E_z(t)$ – see Appendix A. Therefore it is sufficient to calculate the double integral just for the even part of $E_z(t)$, i.e., for

$$E_z(t)_{even} = [E_z(t) - E_z(-t)]/2. \quad (31)$$

Before doing this, we break the integration over the impart parameters in Eq. (29) into the following two parts

$$\sigma(v_z, v_p) = \sigma_1(v_z, v_p) + \sigma_2(v_z, v_p), \quad (32)$$

where

$$\sigma_1(v_z, v_p) = \int_{\rho_0}^{\infty} d\rho\, 2\pi\rho\, [1 - S_a(\rho, v_z, v_p)S_b{}^*(\rho, v_z, v_p)]_\varphi, \quad (33)$$

$$\sigma_2(v_z, v_p) = \int_0^{\rho_0} d\rho\, 2\pi\rho\, [1 - S_a(\rho, v_z, v_p)\, S_b{}^*(\rho, v_z, v_p)]_\varphi. \quad (34)$$

Here

$$\rho_0 = v_p / \omega_B. \quad (35)$$

While expanding $E_z(t)_{even}$ from Eq. (31) in terms of the small parameter $\rho_0/\rho = v_p/(\omega_B\rho)$ and keeping the first non-vanishing term of the expansion, we obtain

$$E_z(t)_{even} = (\sin\varphi)\,(3e\rho v_p v_z/\omega_B)\;\; t[\sin(\omega_B t)]/(\rho^2 + v_z^2t^2)^{5/2}. \tag{36}$$

Then

$$\int_{-\infty}^{\infty} dt\, E_z(t)_{even} \int_{-\infty}^{t} dt_1\, E_z(t_1)_{even} = (1/2)[\int_{-\infty}^{\infty} dt\, E_z(t)]^2 = (\sin^2\varphi)(2e^2\omega_B^{\;2}v_p^{\;2}/v_z^{\;6})\,[K_1(\omega_B\rho/|v_z|)]^2, \tag{37}$$

where $K_1(s)$ is the modified Bessel function of the 2nd kind. After averaging over the phase φ in Eq. (37), we get

$$[1 - S_a(\rho, v_z, v_p)\, S_b^*(\rho, v_z, v_p)]_\varphi = [3X_{\alpha\beta}\,\hbar/(2m_e)]^2\,(\omega_B^{\;2}v_p^{\;2}/v_z^{\;6})\,[K_1(\omega_B\rho/|v_z|)]^2. \tag{38}$$

After substituting the expression (38) into Eq. (33) and integrating over the impact parameters, we obtain

$$\sigma_1(v_z, v_p) = (9\pi^{3/2}/8)\,(X_{\alpha\beta}\hbar/m_e)^2(v_p^{\;2}/v_z^{\;4})\,\mathrm{MeijerG}[\{\{\},\{3/2\}\},\{\{0,0,2\},\{\}\},\, v_p^{\;2}/v_z^{\;2}], \tag{39}$$

where MeijerG[…] is the Meijer G-function.

Now we proceed to calculating $\sigma_2(v_z, v_p)$ defined by Eq. (34). While expanding $E_z(t)_{even}$ from Eq. (31) in terms of the small parameter $\rho/\rho_0 = \omega_B\rho/v_p$ and keeping the first non-vanishing term of the expansion, we obtain

$$E_z(t)_{even} = (\sin\varphi)\,(3e\rho v_p v_z/\omega_B)\; t[\sin(\omega_B t)]/(v_p^{\;2}/\omega_B^{\;2} + v_z^{\;2}t^2)^{5/2}. \tag{40}$$

Then

$$\int_{-\infty}^{\infty} dt\, E_z(t)_{even} \int_{-\infty}^{t} dt_1\, E_z(t_1)_{even} = (1/2)[\int_{-\infty}^{\infty} dt\, E_z(t)]^2 = (\sin^2\varphi)(2e^2\rho^2\omega_B^{\;4}/v_z^{\;6})\{K_1[(v_p^{\;2}/v_z^{\;2})^{1/2}\;]\}^2. \tag{41}$$

After averaging over the phase φ in Eq. (41), we get

$$[1 - S_a(\rho, v_z, v_p)\, S_b^*(\rho, v_z, v_p)]_\varphi = [3X_{\alpha\beta}\hbar/(2m_e)]^2\,(\omega_B^{\;4}\rho^2/v_z^{\;6})\{K_1[(v_p^{\;2}/v_z^{\;2})^{1/2}\;]\}^2. \tag{42}$$

After substituting the expression (42) into Eq. (34) and integrating over the impact parameters, we obtain

$$\sigma_2(v_z, v_p) = (9\pi/8)\,(X_{\alpha\beta}\hbar/m_e)^2(v_p^{\;4}/v_z^{\;6})\,\{K_1[(v_p^{\;2}/v_z^{\;2})^{1/2}\;]\}^2. \tag{43}$$

By combining Eqs. (39) and (43), we get:

$$\sigma(v_z, v_p) = \sigma_1(v_z, v_p) + \sigma_2(v_z, v_p) =$$
$$(9\pi/8)\,(X_{\alpha\beta}\hbar/m_e)^2\,(v_p^{\;2}/v_z^{\;4})\{\pi^{1/2}\mathrm{MeijerG}[\{\{\},\{3/2\}\},\{\{0,0,2\},\{\}\},\, v_p^{\;2}/v_z^{\;2}] + (v_p/v_z)^2K_1[(v_p^{\;2}/v_z^{\;2})^{1/2}\;]^2\}. \tag{44}$$

According to Eq. (28), the adiabatic width in the case of helical trajectories of perturbing electrons is

$$\gamma_{\alpha\beta,hel} = N_e <(v_z^{\;2} + v_p^{\;2})^{1/2}\,\sigma(v_z, v_p)>_{veloc}, \tag{45}$$

where $<\ldots>_{veloc}$ stands for the average over velocities v_p and v_z. In order to get the message across in a relatively uncomplicated form, we simply substitute $v_p^{\;2}$ by its average value over the two dimensional Maxwell distribution $<v_p^{\;2}> = v_T^{\;2}$ and $v_z^{\;2}$ by its average value over the one dimensional Maxwell distribution $<v_z^{\;2}> = v_T^{\;2}/2$. As a result, we obtain the final expression:

$$\gamma_{\alpha\beta,hel} = 7.9(X_{\alpha\beta}\,\hbar/m_e)^2N_e/v_T \text{ or } \gamma_{\alpha\beta,hel}(s^{-1}) = 1.8\text{x}10^{-7}\, X_{\alpha\beta}^{\;2}[N_e(cm^{-3})]/[T_e(eV)]^{1/2},\; v_T = (2T_e/m_e)^{1/2}. \tag{46}$$

It should be noted that $\gamma_{\alpha\beta,hel}$ does not depend on the magnetic field in the case where the magnetic field is strong enough to satisfy the condition (27).

The role of the allowance for helical trajectories of perturbing electrons can be best understood by considering the ratio of $\gamma_{\alpha\beta,hel}$ from Eq. (46) to $\gamma_{\alpha\beta,rec}$ from Eq. (20):

$$\text{ratio} = \gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel} = 0.74/[\ln(1/D) - 0.0772]. \quad (47)$$

Figure 1 shows this ratio versus the dimensionless parameter D, which is defined by Eq. (18) and which is physically the ratio $\rho_{Wad}(v_T)$ / ρ_D. It is seen that for D < 0.44, the allowance for helical trajectories of perturbing electrons *decreases* the adiabatic width, while for D > 0.44, the allowance for helical trajectories of perturbing electrons *increases* the adiabatic width. The fact that, the allowance for helical trajectories of perturbing electrons could lead to two different outcomes (i.e., to either decreasing or increasing the adiabatic width of HDSL) is a *counterintuitive result*.

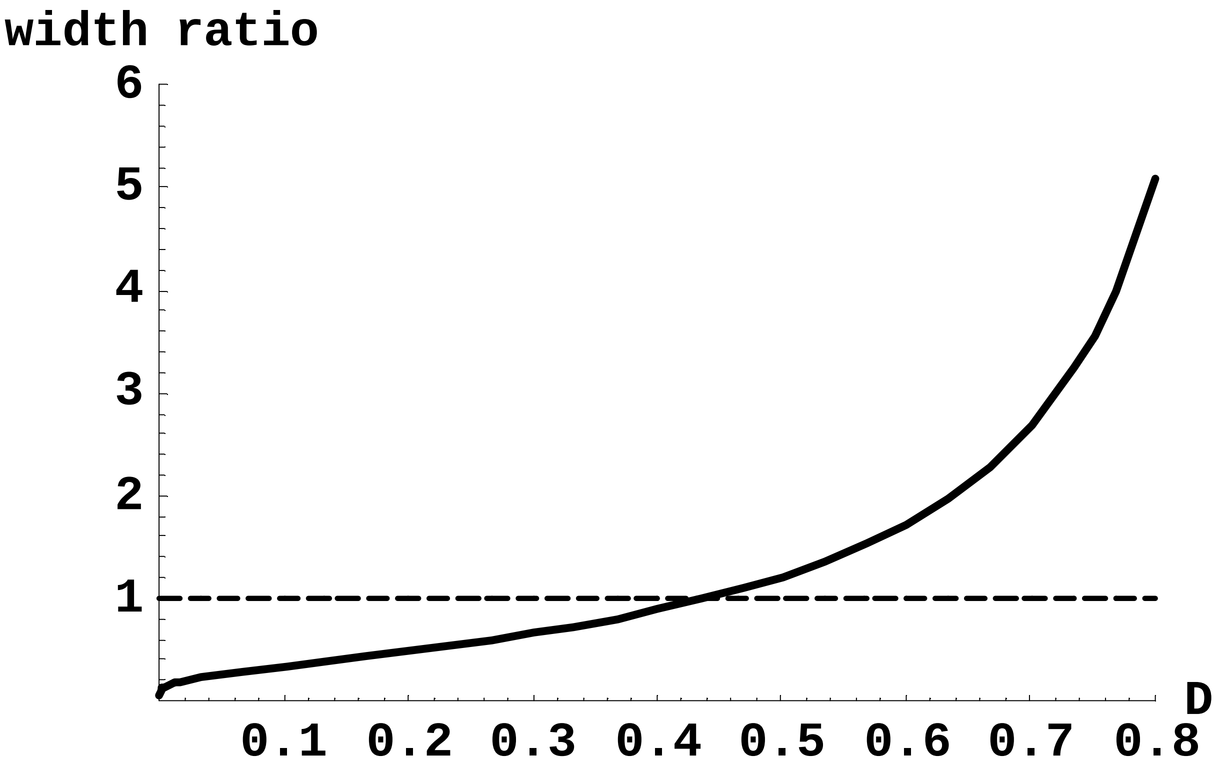

Figure 1: The ratio of adiabatic widths $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ versus the dimensionless parameter D, which is defined by Eq. (18) and which is physically the ratio $\rho_{Wad}(v_T)/\rho_D$ (solid curve). The dashed horizontal line is there for better visualizing the two regions: $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ < 1 and $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ > 1.

Figures 2 shows the ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ from Eq. (47) versus the electron density N_e for the range of N_e relevant to the DA white dwarfs at T_e = 1 eV. The lower curve is for the Lyman-alpha line, for which the adiabatic width is non-zero only for the two π-components of $|X_{\alpha\beta}|$ = 2. The upper curve is for the Balmer-beta line – specifically for its two intense π-components of $|X_{\alpha\beta}|$ = 10.

Figure 3 shows more clearly the ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ for the electron densities below $2\text{x}10^{17}$ cm^{-3} for the Lyman-alpha line (the lower curve) and for the two intense π-components of $|X_{\alpha\beta}|$ = 10 of the Balmer-beta line (the middle curve). The upper curve shows the ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ for the two most intense π-components of $|X_{\alpha\beta}|$ = 28 of the Balmer-delta line.

We also present below explicit practical formulas for the adiabatic width (with the allowance for helical trajectories of perturbing electrons in the case of strong magnetic fields satisfying the condition (27)) in the wavelength scale for the following five HDSL – namely, the Full Width at Half Maximum (FWHM) $\Delta\lambda_{1/2,ad}$.

For Lyman-alpha:

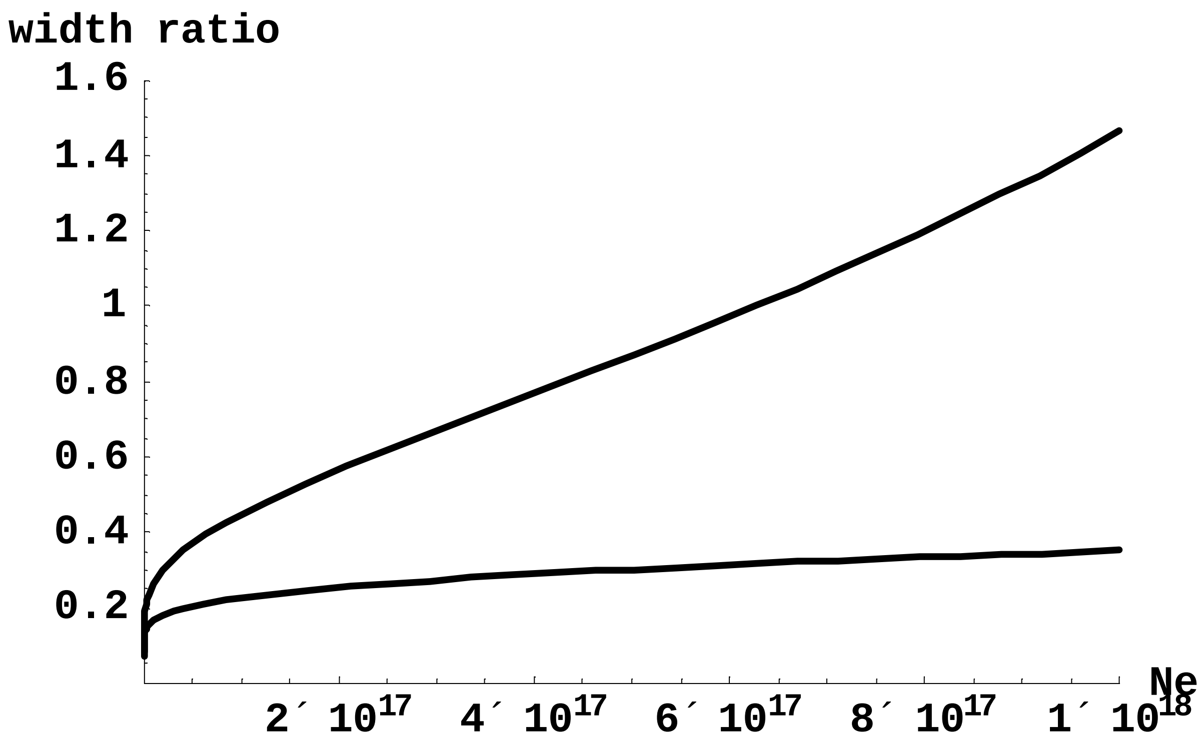

Figure 2. The ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ from Eq. (47) versus the electron density N_e for the range of N_e relevant to the DA white dwarfs at T_e = 1 eV. The lower curve is for the Lyman-alpha line. The upper curve is for the Balmer-beta line – specifically for its two intense π-components of $|X_{\alpha\beta}|$ = 10.

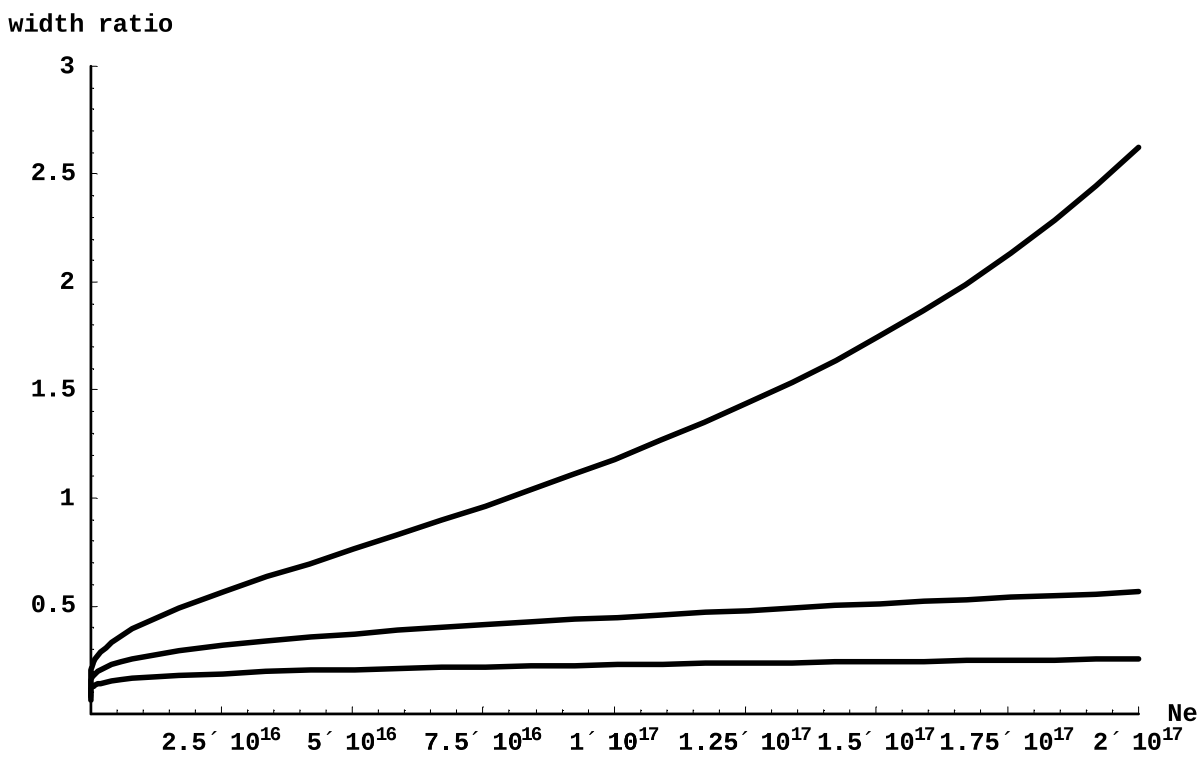

Figure 3: The ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ for the electron densities below $2x10^{17}$ cm^{-3} for the Lyman-alpha line (the lower curve) and for the two intense π-components of $|X_{\alpha\beta}|$ = 10 of the Balmer-beta line (the middle curve). The upper curve shows the ratio $\gamma_{\alpha\beta,hel}/\gamma_{\alpha\beta,hel}$ for the two most intense π-components of $|X_{\alpha\beta}|$ = 28 of the Balmer-delta line.

$$\Delta\lambda_{1/2,ad}(nm) = 1.1x10^{-20} N_e(cm^{-3}) / [T_e(eV)]^{1/2}. \quad (48)$$

For Balmer-alpha line components:

$$\Delta\lambda_{1/2,ad}(nm) = 8.1x10^{-20} X_{\alpha\beta}{}^2 Ne(cm^{-3})/[T_e(eV)]^{1/2}. \quad (49)$$

For Balmer-beta line components:

$$\Delta\lambda_{1/2,ad}(nm) = 4.5x10^{-20} X_{\alpha\beta}{}^2 Ne(cm^{-3}) / [T_e(eV)]^{1/2}. \quad (50)$$

For Balmer-gamma line components:

$$\Delta\lambda_{1/2,ad}(nm) = 3.7x10^{-20} X_{\alpha\beta}{}^2 Ne(cm^{-3})/[T_e(eV)]^{1/2}. \quad (51)$$

For Balmer-delta line components:

$$\Delta\lambda_{1/2,ad}(nm) = 3.2x10^{-20} X_{\alpha\beta}{}^2 Ne(cm^{-3})/[T_e(eV)]^{1/2}. \quad (52)$$

We remind that $X_{\alpha\beta}$ is the combination of the parabolic quantum numbers defined in Eq. (11).

Finally, we note that under the condition (21), the Stark profile of the entire HDSL is simply the sum of Lorentzians corresponding to each Stark component

$$S(\Delta\omega) = \Sigma\, j_{\alpha\beta}\, L_{\alpha\beta}(\Delta\omega),$$

$$L_{\alpha\beta}(\Delta\omega) = \int_0^\infty dF\, W(F)(1/\pi)\, \gamma_{\alpha\beta}/[\gamma_{\alpha\beta}{}^2 + (\Delta\omega - C_{\alpha\beta}F)^2], \quad \gamma_{\alpha\beta} = \gamma_{\alpha\beta,hel} + \gamma_{\alpha\beta,nat},\ C_{\alpha\beta} = 3X_{\alpha\beta}\hbar/(2m_e e). \quad (53)$$

Here $j_{\alpha\beta}$ is the relative intensity of the Stark component labeled "$\alpha\beta$", W(F) is the distribution of the quasistatic field F, $\gamma_{\alpha\beta,nat}$ is the natural (radiative) width of the particular Stark component. The summation is over both π- and σ-components, but for the latter $\gamma_{\alpha\beta,hel} = 0$.

3. CONCLUSIONS

We considered the effect of helical trajectories of perturbing electrons on the width of HDSL for the case of strong magnetic fields, such that the non-adiabatic Stark width practically vanishes and only the adiabatic Stark width remains. Such strong magnetic fields encountered, e.g., in white dwarfs. We calculated analytically the adiabatic Stark width for this case and its ratio to the adiabatic Stark width for the rectilinear trajectories of perturbing electrons. We demonstrated that the adiabatic Stark width calculated with the allowance for helical trajectories of perturbing electrons does not depend on the magnetic field for the case of strong magnetic fields under consideration.

We showed that, depending on the particular HDSL and on plasma parameters, the adiabatic Stark width, calculated with the allowance for helical trajectories of perturbing electrons, can be either *by orders of magnitude smaller*, or of the same order, or several times higher than the adiabatic Stark width, calculated for rectilinear trajectories of perturbing electrons. Such a variety of outcomes is a counterintuitive result. We also demonstrated that for the range of plasma parameters typical for DA white dwarfs (i.e, for white dwarfs emitting hydrogen lines), the neglect for the actual, helical trajectories of perturbing electrons can lead to:

– the *overestimation of the Stark width by up to one order of magnitude* for the alpha- and beta-lines of the Lyman and Balmer series;

– the *underestimation of the Stark width by several times* for the delta- and higher-lines of the Balmer series.

Therefore, our results should motivate astrophysicists for a *very significant revision* of all existing calculations of the broadening of hydrogen lines in DA white dwarfs.

The last but not least: *at any value of the magnetic field* (no matter how large or small), the Stark width of the central (unshifted) component of the Ly-alpha Zeeman triplet has practically only the adiabatic contribution, as shown in detail in paper [4]. So, the experimental/observational studies, for which the effect of helical trajectories of perturbing electrons on the Stark width might be significant, are not limited by white dwarfs, but can be performed in a variety of laboratory and astrophysical plasmas emitting the hydrogen or deuterium Ly-alpha line (by using the polarization analysis).

Appendix A. Proof of the symmetry properties of the double integral in Eq. (30)

We consider the following two double integrals:

$$I = \int_{-\infty}^{\infty} dt\, g(t) \int_{-\infty}^{t} dt_1\, g(t_1), \tag{A.1}$$

$$J = \int_{-\infty}^{\infty} dt\, g(t) \int_{t}^{\infty} dt_1\, g(t_1), \tag{A.2}$$

Obviously

$$I + J = \left[\int_{-\infty}^{\infty} dt\, g(t)\right]^2. \tag{A.3}$$

If the function g(t) is even, so that g(–t) = g(t), then after substituting u = –t, the integral J becomes

$$J = \int_{-\infty}^{\infty} du\, g(u) \int_{-u}^{\infty} dt_1\, g(t_1). \tag{A.4}$$

After further substituting $w = -t_1$, the integral J finally yields

$$J = \int_{-\infty}^{\infty} du\, g(u) \int_{-\infty}^{\infty} dw\, g(w) = I, \tag{A.5}$$

so that

$$I = (I + J)/2 = (1/2)[\int_{-\infty}^{\infty} dt\, g(t)]^2. \tag{A.6}$$

If the function g(t) is odd, so that g(–t) = –g(t), then after substituting u = –t and the subsequent substation w = –t, we would still get

$$I = (I + J)/2 = (1/2)[\int_{-\infty}^{\infty} dt\, g(t)]^2. \tag{A.7}$$

However, for the odd function g(t), the integral in the right side of Eq. (A.7) is zero.

References

[1] E. Oks, J. Quant. Spectrosc. Rad. Transfer **171**, (2016) 15.

[2] P. Kepple and H.R. Griem, Phys. Rev. **173** (1968) 317.

[3] H.R. Griem, *Spectral Line Broadening by Plasmas* (Academic, New York) 1974.

[4] A. Derevianko and E. Oks, Phys. Rev. Letters **73** (1994) 2059.

[5] Oks, 2017, J. Phys. Conf. Ser. **810** (2017) 012006.

[6] V.S. Lisitsa, Sov. Phys. Uspekhi **122** (1977) 603.

[7] Ya. Ispolatov and E. Oks, J. Quant. Spectr. Rad. Transfer **51** (1994) 129.

[8] E. Oks, J. Quant. Spectrosc. Rad. Transfer **152** (2015) 74.

[9] E. Oks, *Diagnostics of Laboratory and Astrophysical Plasmas Using Spectral Lines of One-, Two-, and Three-Electron Systems* (World Scientific, New Jersey) 2017.

[10] B. Franzon and St. Schramm, J. Phys. Conf. Ser. **861** (2017) 012015.

[11] D. Reimers, S. Jordan, D. Koester, and N. Bade, Astron. Astrophys. **311** (1996) 572.